Experimenting with Grammars to Generate L-Systems – in JFLAP
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L-Systems

- Grammatical systems introduced by Lyndonmayer
- Model biological systems and create fractals
- Similar to Chomsky grammars, except all variables are replaced in each step, not just one!
- Successive strings are interpreted as strings of render commands and displayed graphically
Parts of an L-System (a type of grammar)

• Defined over an alphabet

• Three parts
  – Axiom (starting place)
  – Replacement rules (replaces all variables at once)
  – Geometric rules (for drawing)
    • $g$ means move forward one unit with pen down
    • $f$ means move forward one unit with pen up
    • $+$ means turn right by the default angle
    • $-$ means turn left by the default angle
L-System

An L-system is composed of three parts \((\Sigma, h, w)\)

\[\Sigma\] finite alphabet set of symbols

\[h\] rewriting rules each symbol is replaced by string of symbols

\[w\] axiom starting point

\[h\] is finite substitutions, \(h: \Sigma \rightarrow \Sigma^*\).
\( h(w) \)

\( h(w) \) is computed by replacing every symbol in \( w \) that has a rewrite rule by that rule.

A language \( L \) of an L-system is the word sequence generated by

\[
\begin{align*}
\bullet & \quad h^0(w) = w \\
\bullet & \quad h^1(w) = h(w) \\
\bullet & \quad h^2(w) = h(h(w)) \\
\bullet & \quad \ldots
\end{align*}
\]

\[ L = \{ h^i(w) \mid i \geq 0 \} \]
NOTE: If $h(a)=bb$ we will write this as a rule

$$a \rightarrow bb$$
Example:

$\Sigma$ alphabet: $\{a, b\}$  

h rules: $a \rightarrow aa$  

$\qquad b \rightarrow ab$  

w axiom: $ab$

What is the language $L$ of strings represented by this $L$-system?

$L = \{ab, aabb, aaaaaaaabb, \ldots \}$
Drawing a picture of an L-system

Defining an L-system: (3 parts in this order)

- Axiom definition: This must be the first line of the file
- Production rules: Defines the replacement rules.
- Geometric rules: Defines colors, widths, etc.
Graphically represent

Symbols for drawing and moving:

- **g**: draw a line one step in the current direction
- **f**: move forward one step in the current direction
Example: example1

axiom X

X -> g f g X

distance 15
lineWidth 5
color black

L = \{ X, g(X), g(g(X)) \}

What does this draw?
Geometric rules

• +  change direction to the right
• −  change direction to the left
• %  change direction 180 degrees
• ~  decrement the width of the next lines
• [  save in stack current state info
• ]  recover from stack state info
• {  start filled in polygon
• }  end filled in polygon
Example – lsys-samp1

- Axiom

- Replacement Rules

- Geometric Rules

NOTE: Must use spaces as separator between symbols
Example – lsys-samp1 (cont)

• Derivation of strings

\[ \text{Note: replace both } X \text{ and } Y \text{ each time} \]
Example – lsys-samp2

Axion: X

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>lineWidth</td>
<td>4</td>
</tr>
<tr>
<td>lineIncrement</td>
<td>1</td>
</tr>
<tr>
<td>distance</td>
<td>18</td>
</tr>
<tr>
<td>color</td>
<td>black</td>
</tr>
<tr>
<td>angle</td>
<td>30</td>
</tr>
</tbody>
</table>
Example – lsys-samp2 (cont)

\[ g[\sim+Yg]gX \]

\[ g[\sim++Yg]gg[\sim+Yg]gX \]

\[ g[\sim+++Yg]gg[\sim++Yg]gg[\sim+Yg]gX \]

...
Example - tree

![Diagram showing tree structure in JFLAP](image)

Axiom: $R \sim \#\# B$

<table>
<thead>
<tr>
<th>Character</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>$\rightarrow [\sim ## T L - B + + B]$</td>
</tr>
<tr>
<td>L</td>
<td>$\rightarrow [\sim - g + + g % -- g]$</td>
</tr>
<tr>
<td>R</td>
<td>$\rightarrow ! @@ R$</td>
</tr>
<tr>
<td>T</td>
<td>$\rightarrow T g$</td>
</tr>
</tbody>
</table>

**Name** | **Parameter**
---|---
color | brown
polygonColor | forestGreen
Example – tree rendered
Stochastic Tree

- Add a rule $T \rightarrow T$

- Now there is a choice for $T$, draw a line or don’t
Same Stochastic L-System

• Rendered 3 times, each at 8\textsuperscript{th} derivation
JFLAP

- JFLAP is available for free:
  www.jflap.org
- Duke School of Environment uses L-systems to model pine needles in Duke Forest
Classwork 6- Exercise 1

- Write an L-system for the picture below.
- Symbols needed are: g, + and one variable
- Distance of the line is 100, rendering at 1 draws the first line, each additional render draws another line.

[Diagram of a geometric pattern]
Exercise 2

• Write an L-system for the picture below.
• Symbols may need: g, %, +
• Distance set to 15, angle set to 45, side of square is length 30, first diagonal line is 60
• 1\text{st}, 2\text{nd} and 6\text{th} renderings shown
Exercise 3

- Write an L-system for the picture below.
- Symbols may need: g, +, -, [ ]
- Angle set to 90, distance set to 15
- Shows 1\textsuperscript{st}, 2\textsuperscript{nd} and 3\textsuperscript{rd} renderings