Section: Turing Machines - Building Blocks

1. Given Turing Machines M1 and M2
Notation for

- Run M1
- Run M2

\[ \rightarrow M1 \rightarrow M2 \]

\[ S \rightarrow S' \]
\[ H \rightarrow H' \]

\[ z;R \rightarrow z;L \]

\[ z \text{ represents any symbol in } \]

\[ \square \]
2. Given Turing Machines M1 and M2

M1

M2

\[ \rightarrow M1 \xrightarrow{x} M2 \]

\[ \rightarrow S \quad H \quad \rightarrow S' \quad H' \]

\[ \rightarrow S \quad H \quad \rightarrow x; z; R \quad \rightarrow z; z; L \]

z represents any symbol in
x is an element of
3. Given Turing Machines M1, M2, and M3

- M1
  - S
  - H

- M2
  - S'
  - H'

- M3
  - S''
  - H''

- x is an element of S
- y is any element except x from H
- z is any element from S'
More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

$z$ is any symbol in $\Gamma$

$x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right
3. $L$ - move left
4. $x$ - write $x$ (and don’t move)
5. $R_a$ - move right until you see an $a$
6. $L_a$ - move left until you see an $a$

7. $R_a$ - move right until you see anything that is not an $a$

8. $L_a$ - move left until you see anything that is not an $a$

9. $h$ - halt in a final state

10. $\{a, b\} \rightarrow w$

   If the current symbol is $a$ or $b$, let $w$ represent the current symbol.
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}$. If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb

input: ba, output: ba

What is the running time?

$|w| = n$
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}$, $|w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: abbabb, output: abbabbbb

The tape head should finish pointing at the leftmost symbol of $w$. 
Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function $f:D \rightarrow R$ is a TM $M$, which given input $d \in D$, halts with answer $f(d) \in R$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

\[
\begin{align*}
\text{start with:} & \quad 111 + 1111 \\
\text{end with:} & \quad 1111111
\end{align*}
\]
Example: Copy a String, \( f(w) = w0w \), \( w \in \Sigma^* \), \( \Sigma = \{a, b, c\} \)

Denoted by \( C \)

\[
\begin{align*}
\text{start with:} & \quad \text{abac} \\
\uparrow & \\
\text{end with:} & \quad \text{abac0abac} \\
\uparrow & \\
\end{align*}
\]

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right).

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with: $\underline{aaBbaba}c_\text{a}$

end with: $\underline{aaBBbaca}$

\[ \text{In part you are shifting } l_w = n \]

\[ \Theta(n) \]
Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with: $\text{babcaBba}$

end with: $\text{bacaBBba}$

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, \( f(x \cdot y) = x \cdot y \), \( x \) and \( y \) unary numbers. Assume \( x,y > 0 \).

\[
\begin{align*}
\text{start with:} & \quad 1111 \times 11 \\
& \quad \uparrow \\
\text{end with:} & \quad 11111111 \\
& \quad \uparrow
\end{align*}
\]