Ch. 7 - Pushdown Automata

A DFA = \( (Q, \Sigma, \delta, q_0, F) \)
Modify DFA by adding a stack. New machine is called Pushdown Automata (PDA).
Definition: Nondeterministic PDA (NPDA) is defined by

\[ M = (Q, \Sigma, \Gamma, \delta, q_0, z, F) \]

where

- \( Q \) is finite set of states
- \( \Sigma \) is tape (input) alphabet
- \( \Gamma \) is stack alphabet
- \( q_0 \) is initial state
- \( z \) - start stack symbol, \( z \in \Gamma \)
- \( F \subseteq Q \) is set of final states.

\[ \delta : Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \to \text{finite subsets of } Q \times \Gamma^* \]
Example of transitions

$\delta(q_1,a,b) = \{(q_3,b), (q_4,ab), (q_6,\lambda)\}$

The diagram for the above transitions is:

![Diagram of transitions](image-url)
Instantaneous Description:

\((q,w,u)\)

Description of a Move:

\((q_1,aw,bx) \vdash (q_2,w,yx)\)

iff

\((q_2,y) \in \delta(q_1,a,b)\)

Definition Let \(M=(Q,\Sigma,\Gamma,\delta,q_0,z,F)\) be a NPDA. \(L(M)=\{w \in \Sigma^* \mid (q_0,w,z) \vdash^* (p,\lambda,u), p \in F, u \in \Gamma^*\}\). The NPDA accepts all strings that start in \(q_0\) and end in a final state.
Example: $L = \{a^n b^n | n \geq 0\}$, $\Sigma = \{a, b\}$,
$\Gamma = \{z, a\}$
Another Definition for Language Acceptance

NPDA $M$ accepts $L(M)$ by empty stack:

$$L(M) = \{ w \in \Sigma^* | (q_0, w, z)^* \vdash (p, \lambda, \lambda) \}$$
Example: $L = \{a^n b^m c^{n+m} | n, m > 0\}$, 
$\Sigma = \{a, b, c\}$, $\Gamma = \{0, z\}$
Examples for you to try on your own: (solutions are at the end of the handout).

- $L = \{a^n b^m | m > n, m, n > 0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a\}$
- $L = \{a^n b^{n+m} c^m | n, m > 0\}$, $\Sigma = \{a, b, c\}$
- $L = \{a^n b^{2n} | n > 0\}$, $\Sigma = \{a, b\}$
Definition: A PDA $M=(Q,\Sigma,\Gamma,\delta,q_0,z,F)$ is deterministic if for every $q \in Q$, $a \in \Sigma \cup \{\lambda\}$, $b \in \Gamma$

1. $\delta(q, a, b)$ contains at most 1 element
2. if $\delta(q, \lambda, b) \neq \emptyset$ then $\delta(q, c, b) = \emptyset$ for all $c \in \Sigma$

Definition: $L$ is DCFL iff $\exists$ DPDA $M$ s.t. $L=L(M)$. 
Examples:

1. Previous pda for $\{a^n b^n | n \geq 0\}$ is deterministic?

2. Previous pda for $\{a^n b^m c^{n+m} | n, m > 0\}$ is deterministic?

3. Previous pda for $\{w w^R | w \in \Sigma^+ \}, \Sigma = \{a, b\}$ is deterministic?
Example: \( L = \{a^n b^m | m > n, m, n > 0\} \), 
\( \Sigma = \{a, b\} \), \( \Gamma = \{z, a\} \)

Example: \( L = \{a^n b^{n+m} c^m | n, m > 0\} \), 
\( \Sigma = \{a, b, c\} \),

Example: \( L = \{a^n b^{2n} | n > 0\} \), \( \Sigma = \{a, b\} \)