Section: Turing Machines - Building Blocks

1. Given Turing Machines M1 and M2

Notation for

- Run M1
- Run M2

\[
\begin{align*}
M1 & \quad M2 \\
S & \quad H & \quad S' & \quad H'
\end{align*}
\]

\[
\begin{align*}
\rightarrow & \quad \text{M1} & \quad \rightarrow & \quad \text{M2}
\end{align*}
\]

\[
\begin{align*}
S & \quad H & \quad z; z, \text{R} & \quad z; z, \text{L} & \quad S' & \quad H'
\end{align*}
\]

\[
z \text{ represents any symbol in }
\]

2. Given Turing Machines M1 and M2

M1

M2

$\rightarrow M1 \xrightarrow{x} M2$

$\rightarrow S \quad H \quad \rightarrow S' \quad H'$

$\rightarrow S \quad H \xrightarrow{x; x, R} z; z, L \rightarrow S' \quad H'$

$z$ represents any symbol in

$x$ is an element of
3. Given Turing Machines M1, M2, and M3

M1

\[ \rightarrow S \quad H \]

M2

\[ \rightarrow S' \quad H' \]

M3

\[ \rightarrow S'' \quad H'' \]

x\rightarrow M2

M1 \rightarrow x \rightarrow M3

\[ \begin{align*}
x;x,R & \rightarrow z;z,L \\
y;y,R & \rightarrow z;z,L \\
z;z,L & \rightarrow S'' \quad H''
\end{align*} \]

x is an element of

y is any element except x from

z is any element from
More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

- $z$ is any symbol in $\Gamma$
- $x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right
3. $L$ - move left
4. $x$ - write $x$ (and don’t move)
5. $R_a$ - move right until you see an $a$
6. $L_a$ - move left until you see an \( a \)

7. $R_{\neg a}$ - move right until you see anything that is not an \( a \)

8. $L_{\neg a}$ - move left until you see anything that is not an \( a \)

9. \( h \) - halt in a final state

10. \( \{a,b\} \to w \)

   If the current symbol is \( a \) or \( b \), let \( w \) represent the current symbol.
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$. If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb
input: ba, output: ba

What is the running time?
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$, $|w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

Input: $abbabb$, Output: $abbbabbb$

The tape head should finish pointing at the leftmost symbol of $w$. 
Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An \textit{algorithm} for a function $f: D \rightarrow R$ is a TM $M$, which given input $d \in D$, halts with answer $f(d) \in R$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

\begin{align*}
\text{start with:} & \quad 111 + 1111 \\
\uparrow & \\
\text{end with:} & \quad 1111111 \\
\uparrow &
\end{align*}
Example: Copy a String, $f(w) = w0w$, $w \in \Sigma^*$, $\Sigma = \{a, b, c\}$

Denoted by $C$

start with: abac

end with: abac0abac

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

\[
\begin{align*}
\text{start with:} & \quad \text{aaBbabc} \\
\uparrow & \\
\text{end with:} & \quad \text{aaBBbaca}
\end{align*}
\]
Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with: babcaBba

↑

end with: bacaBBba

↑

(similar to $S_R$)
\[
\begin{align*}
\text{s L} \xrightarrow{a,b,c,B} \{v\} \xrightarrow{\nu} 0 \\
\rightarrow R B R \xrightarrow{a,b,c} \{w\} \xrightarrow{\omega} B L w R \\
\rightarrow B \\
\rightarrow L 0 \xrightarrow{v} R h
\end{align*}
\]
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, \( f(x*y) = x*y \), \( x \) and \( y \) unary numbers. Assume \( x,y > 0 \).

\[
\begin{align*}
\text{start with:} & \quad 1111 \times 11 \\
& \uparrow \\
\text{end with:} & \quad 11111111 \\
& \uparrow 
\end{align*}
\]