DFA = (Q, Σ, δ, q_0, F)

where

Q is finite set of states
Σ is tape (input) alphabet
q_0 is initial state
F ⊆ Q is set of final states.
δ : Q × Σ → Q
Example: DFA that accepts even binary numbers.

Transition Diagram:

\[ M = (Q, \Sigma, \delta, q_0, F) = \]

Tabular Format

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<thead>
<tr>
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<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
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<td>q1</td>
<td></td>
</tr>
<tr>
<td>q1</td>
<td>q0</td>
<td></td>
</tr>
</tbody>
</table>

Example of a move: \( \delta(q_0, 1) = \)
Algorithm for DFA:

Start in start state with input on tape
q = current state
s = current symbol on tape
while (s != blank) do
    q = $\delta(q,s)$
    s = next symbol to the right on tape
if q $\in$ F then accept

Example of a trace: 11010
Pictorial Example of a trace for 100:

1) \[
\begin{array}{c}
1 \\
0 \\
0 \\
\end{array}
\]

2) \[
\begin{array}{c}
1 \\
0 \\
0 \\
\end{array}
\]

3) \[
\begin{array}{c}
1 \\
0 \\
0 \\
\end{array}
\]

4) \[
\begin{array}{c}
1 \\
0 \\
0 \\
\end{array}
\]
Definition:
\[ \delta^*(q, \lambda) = q \]
\[ \delta^*(q, wa) = \delta(\delta^*(q, w), a) \]

Definition The language accepted by a DFA \( M = (Q, \Sigma, \delta, q_0, F) \) is set of all strings on \( \Sigma \) accepted by \( M \). Formally, \[ L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \in F\} \]
Trap State

Example: \( L(M) = \)
Example:

$L = \{ w \in \Sigma^* \mid w \text{ has an even number of a’s and an even number of b’s} \}$
Example: DFA that accepts even binary numbers that have an even number of 1’s.
Definition A language is regular iff there exists DFA $M$ s.t. $L = L(M)$. 
Chapter 2.2

Nondeterministic Finite Automata (or Accepter)

Definition

An NFA = (Q, Σ, δ, q₀, F)

where

Q is finite set of states
Σ is tape (input) alphabet
q₀ is initial state
F ⊆ Q is set of final states.

δ: Q × (Σ ∪ {λ}) → 2^Q
Example

Note: In this example $\delta(q_0, a) =$

$L =$
Example

\[ L = \{(ab)^n \mid n > 0\} \cup \{a^n b \mid n > 0\} \]
Definition $q_j \in \delta^*(q_i, w)$ if and only if there is a walk from $q_i$ to $q_j$ labeled $w$.

Example From previous example:

$\delta^*(q_0, ab) =$

$\delta^*(q_0, aba) =$

Definition: For an NFA $M$, 
$L(M) = \{w \in \Sigma^* | \delta^*(q_0, w) \cap F \neq \emptyset\}$
2.3 NFA vs. DFA: Which is more powerful?

Example:
Theorem Given an NFA 
\( M_N = (Q_N, \Sigma, \delta_N, q_0, F_N) \), then there 
eexists a DFA \( M_D = (Q_D, \Sigma, \delta_D, q_0, F_D) \) 
such that \( L(M_N) = L(M_D) \).

Proof:

We need to define \( M_D \) based on \( M_N \).

\( Q_D = \) 
\( F_D = \) 
\( \delta_D : \)
Algorithm to construct $M_D$

1. start state is $\{q_0\} \cup \text{closure}(q_0)$

2. While can add an edge
   (a) Choose a state $A=\{q_i, q_j, ... q_k\}$ with missing edge for $a \in \Sigma$
   (b) Compute $B = \delta^*(q_i, a) \cup \delta^*(q_j, a) \cup ... \cup \delta^*(q_k, a)$
   (c) Add state $B$ if it doesn’t exist
   (d) add edge from $A$ to $B$ with label $a$

3. Identify final states

4. if $\lambda \in L(M_N)$ then make the start state final.
Example:
Properties and Proving - Problem 1

Consider the property
Replace_one_a_with_b or R1awb for short. If L is a regular, prove R1awb(L) is regular.

The property R1awb applied to a language L replaces one a in each string with a b. If a string does not have an a, then the string is not in R1awb(L).

Example 1: Consider L={aaab, bbba}

R1awb(L)=

Example 2: Consider \( \Sigma = \{a, b\} \), L = \( \{w \in \Sigma^* \mid w \) has an even number of a’s and an even number of b’s\}

R1awb(L)=

Proof:
Properties and Proving - Problem 2
Consider the property
Truncate_all_preceeding_b’s or
TruncPreb for short. If L is a regular,
prove TruncPreb(L) is regular.
The property TruncPreb applied to a
language L removes all preceeding b’s
in each string. If a string does not
have an preceeding b, then the string
is the same in TruncPreb(L).
Example 1: Consider L={aaab, bbba}
TruncPreb(L)=
Example 2: Consider L =
{(bba)^n | n > 0}
TruncPreb(L)=
Proof:
Minimizing Number of states in DFA

Why?

Algorithm

• Identify states that are indistinguishable
  These states form a new state

Definition Two states \( p \) and \( q \) are indistinguishable if for all \( w \in \Sigma^* \)

\[
\begin{align*}
\delta^*(q, w) \in F & \Rightarrow \delta^*(p, w) \in F \\
\delta^*(p, w) \notin F & \Rightarrow \delta^*(q, w) \notin F
\end{align*}
\]

Definition Two states \( p \) and \( q \) are distinguishable if \( \exists w \in \Sigma^* \) s.t.

\[
\begin{align*}
\delta^*(q, w) \in F & \Rightarrow \delta^*(p, w) \notin F \text{ OR} \\
\delta^*(q, w) \notin F & \Rightarrow \delta^*(p, w) \in F
\end{align*}
\]
Example:
Example: