Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

```
<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>b</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>tape head</td>
<td>head moves</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

current state

```
0 1 2 3 4 5
```

Turing Machine:

```
<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>b</th>
<th>a</th>
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</tbody>
</table>
```

current state

```
0 1 2 3 4 5
```
Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
  - tape
- actions
  - write symbol
  - read symbol
  - move left (L) or right (R)
- computation
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

Formal Definition of TM

A TM $M$ is defined by $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ where

- $Q$ is finite set of states
- $\Sigma$ is input alphabet
- $\Gamma$ is tape alphabet
- $B \in \Gamma$ is blank
- $q_0$ is start state
- $F$ is set of final states
- $\delta$ is transition function

$\delta(q,a) = (p,b,R)$ means “if in state $q$ with the tape head pointing to an ’a’, then move into state $p$, write a ’b’ on the tape and move to the right”.

**TM as Language recognizer**

**Definition:** Configuration is denoted by ⊢.

if δ(q,a) = (p,b,R) then a move is denoted

abaqabba ⊢ ababpbba

**Definition:** Let M be a TM, M=(Q,Σ,Γ,δ,q₀,B,F). L(M) = {w ∈ Σ*|q₀w ⊢ x₁q_fx₂ for some q_f ∈ F, x₁, x₂ ∈ Γ*}

**TM as language acceptor**

M is a TM, w is in Σ*,

- if w ∈ L(M) then M halts in final state
- if w ∉ L(M) then either
  - M halts in non-final state
  - M doesn’t halt

**Example**

Σ = {a, b}

Replace every second ‘a’ by a ‘b’ if string is even length.

- Algorithm
Example:

$L = \{ a^n b^n c^n | n \geq 1 \}$

Is the following TM Correct?

In a transducer, TM can implement a function $f(w) = w'$

<table>
<thead>
<tr>
<th>Start</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>$w'$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transition</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>a;1,R</td>
<td>b;2,R</td>
</tr>
<tr>
<td>a;a,R</td>
<td>b;b,R</td>
</tr>
<tr>
<td>2;2,R</td>
<td>3;3,R</td>
</tr>
<tr>
<td>c;3,L</td>
<td>a;a,L</td>
</tr>
<tr>
<td>b;b,L</td>
<td>2;2,L</td>
</tr>
<tr>
<td>3;3,L</td>
<td>1;1,R</td>
</tr>
</tbody>
</table>
**Definition:** A function with domain D is *Turing-computable* or *computable* if there exists TM $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ such that

$$q_0 w \xrightarrow{*} q_f f(w)$$

$q_f \in F$, for all $w \in D$.

**Example:**

$f(x) = 2x$

$x$ is a unary number

- **Start with:** 111
- **End with:** 111111

Is the following TM correct?
Example:

\[ L = \{ww \mid w \in \Sigma^+ \}, \Sigma = \{a, b\} \]