CompSci 116: Lecture 12: Decisions

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April 11, 2019
"We asked 20 house officers, 20 fourth-year medical students and 20 attending physicians, selected in 67 consecutive hallway encounters at four Harvard Medical School teaching hospitals, the following question:

"If a test to detect a disease whose prevalence is 1/1000 has a false positive rate of 5%, what is the chance that a person found to have a positive result actually has the disease, assuming that you know nothing about the person's symptoms or signs?"
Eleven of 60 participants, or 18%, gave the correct answer. These participants included four of 20 fourth-year students, three of 20 residents in internal medicine and four of 20 attending physicians. The most common answer, given by 27, was that [the chance that a person found to have a positive result actually has the disease] was 95%.
Probability Review

- P(A): Probability of event A
  - Value in [0,1]
  - How likely is A to happen?
- P(A) + P(not A) = 1
- P(A ∪ B) = P(A) + P(B) − P(A ∩ B)
  
  e.g., x = roll of a 6-sided die;
  
P(x is even ∪ x > 3) 
= P(x is even) + P(x > 3) − P(x is even ∩ x > 3) 
= 0.5 + 0.5 − 1/3 = 2/3
Assume 500 students enrolled in both Calculus and Physics. Of these students,

- 82 got an A in calculus,
- 73 got an A in physics, and
- 42 got an A in both courses.

Which of the following probabilities is the smallest?

The probability that a randomly chosen student:

a. Got an A in at least one of the two courses
b. Got less than an A in at least one of the two courses
c. Got an A in both courses
d. Got an A in calculus but not in physics
e. Got an A in physics but not calculus

From Derek Bruff, Vaanderbilt
Round One

- Scenario:
  - Class consists of second years (60%) and third years (40%)
  - 50% of the second years have declared their major
  - 80% of the third years have declared their major
  - I pick one student at random.

- Which is more likely: Second year or third year?
  - Second year, because they are 60% of the class
Round Two

- Slightly different scenario:
  - Class consists of second years (60%) and third years (40%)
  - 50% of the second years have declared their major
  - 80% of the third years have declared their major
  - I pick one student at random... 
    That student has declared a major!

- Second Year or Third Year?
Terminology

Prior probability

\[ P(B \mid A) = \frac{P(A \cap B)}{P(A)} \]

Likelihood/Conditional probability

Likelihood
Problem 2: Conditional Probability

- Hat with three cards,
  - Blue on both sides
  - Red on both sides
  - Red/Blue
- A card is drawn at random from the hat and you see that one side is blue

What is the probability the other side is blue?

- a. \( \frac{1}{3} \)
- b. \( \frac{1}{2} \)
- c. \( \frac{2}{3} \)
Bayes’ Rule

Pick a student at random.

Posterior probability:

\[
P(\text{Third Year} \mid \text{Declared}) = \frac{0.4 \times 0.8}{(0.6 \times 0.5) + (0.4 \times 0.8)}
\]

\[
= \frac{0.5161...}{0.5161...}
\]

= 0.5161...
Purpose of Bayes’ Rule

- Update your prediction based on new information
- In a multi-stage experiment, find the chance of an event at an earlier stage, given the result of a later stage

(Demo)
Example: Doctors & Clinical Tests

- Out of 1000 patients:

<table>
<thead>
<tr>
<th></th>
<th>Positive test result</th>
<th>Negative test result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has disease</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Doesn't have disease</td>
<td>49.95</td>
<td>949.05</td>
</tr>
</tbody>
</table>

- So only $\frac{1}{50.95}$ of patients with positive test results have the disease.