Programming Assignment 2.5: Estimating Utilities (due April 6 before class)

Please read the rules for assignments on the course web page [http://www.cs.duke.edu/courses/spring18/compsci223/]. Use Piazza (preferred) or directly contact Harsh (harsh.parikh@duke.edu), Hanrui (hrzhang@cs.duke.edu), or Vince (conitzer@cs.duke.edu) with any questions. Please use clear variable names and write comments in your code where appropriate (you can put comments between /* and */, or start a line with #).

Please see Homework 1 for details about getting set up with GLPK, making a directory for this homework, etc.

Note: in this assignment, there is an example instance that you are asked to solve. However, just getting this example right is not enough to get full credit: your formulation should work on all instances. The example is just there to give you something to test your formulation on.

1. **(Estimating utilities (50 points).)**

Elena Umberta Massima is an expected utility maximizer. When presented with two probability distributions over a set of possible outcomes, E.U.M. says, without hesitation, which she prefers, and you will not catch her in any inconsistencies.

We have four outcomes: A, B, C, D. We will accordingly represent probability distributions as vectors of four probabilities of the respective outcomes. \((p_A, p_B, p_C, p_D) > (p'_A, p'_B, p'_C, p'_D)\) will denote that E.U.M. prefers distribution \(p\) to \(p'\). We learn the following four preferences:

- \((.1, .2, .3, .4) > (.1, .2, .4, .3)\)
- \((.4, .4, .1, .1) > (.4, .2, .2, .2)\)
- \((.6, .1, 0, .3) > (.4, .3, .3, 0)\)
- \((.4, .3, .2, .1) > (.5, .5, 0, 0)\)

Obviously, we jump on the opportunity to estimate the utilities (for outcomes A, B, C, D) of this fascinating woman.
Our goal will be to assign utilities in the interval $[0, 1]$ to the four outcomes that are consistent with E.U.M.'s preferences. Write a linear program formulation for this. You should add an objective to satisfy the consistency constraints by as large a margin as possible (similar to our linear program for strict dominance by mixed strategies). You should use the MathProg (.mod) language to model the general problem (you should allow for more than four outcomes and more than four preferences) and solve the specific instance above. (Hint: the optimal objective value is 0.02.)