Lecture 10a:
Join Algorithms: Nested-Loop Joins
Notation

- Relations: $R$, $S$
- Tuples: $r$, $s$
- Number of tuples: $|R|$, $|S|$
- Number of disk blocks: $B(R)$, $B(S)$
- Number of memory blocks available: $M$
- Cost metric
  - Number of I/O’s
  - Memory requirement

Remember our disk-memory diagram?
Nested-loop join

\[ R \bowtie_p S \]

- For each block of \( R \), and for each \( r \) in the block:
  - For each block of \( S \), and for each \( s \) in the block:
    - Output \( rs \) if \( p \) evaluates to true over \( r \) and \( s \)
- \( R \) is called the outer table; \( S \) is called the inner table
- I/O’s: \( B(R) + |R| \cdot B(S) \)
- Memory requirement: 3

Improvement: block-based nested-loop join
Block-based Nested Loop Join

- $R \bowtie_p S$
- R outer, S inner
- For each block of $R$, for each block of $S$: For each $r$ in the $R$ block, for each $s$ in the $S$ block: ...
  - I/O’s: $B(R) + B(R) \cdot B(S)$
  - Memory requirement: same as before
More improvements

• Make use of available memory
  • Stuff memory with as much of $R$ as possible, stream $S$ by, and join every $S$ tuple with all $R$ tuples in memory
  • I/O’s: $B(R) + \left[ \frac{B(R)}{M-2} \right] \cdot B(S)$
    • Or, roughly: $B(R) \cdot B(S)/M$
  • Memory requirement: $M$ (as much as possible)

• Which table would you pick as the outer?
Lecture 10b: External Sorting
External merge sort

Remember (internal-memory) merge sort?
Problem: sort $R$, but $R$ does not fit in memory

- Number of tuples: $|R|$
- Number of disk blocks: $B(R)$
- Number of memory blocks available: $M$

To Understand:
What is a run?
What is a pass (or level)?

Reminder: How does the 2-way merge sort work?
How to extend to multi-way merge sort?
External merge sort

Remember (internal-memory) merge sort? Problem: sort $R$, but $R$ does not fit in memory

- **Pass 0**: read $M$ blocks of $R$ at a time, sort them, and write out a level-0 run

- **Pass 1**: merge $(M - 1)$ level-0 runs at a time, and write out a level-1 run

- **Pass 2**: merge $(M - 1)$ level-1 runs at a time, and write out a level-2 run

...  

- **Final pass** produces one sorted run
Toy example

• 3 memory blocks available; each holds one number
• Input: 1, 7, 4, 5, 2, 8, 3, 6, 9
• Pass 0
  • 1, 7, 4 → 1, 4, 7
  • 5, 2, 8 → 2, 5, 8
  • 9, 6, 3 → 3, 6, 9
• Pass 1
  • 1, 4, 7 + 2, 5, 8 → 1, 2, 4, 5, 7, 8
  • 3, 6, 9
• Pass 2 (final)
  • 1, 2, 4, 5, 7, 8 + 3, 6, 9 → 1, 2, 3, 4, 5, 6, 7, 8, 9
Analysis

- **Pass 0**: read $M$ blocks of $R$ at a time, sort them, and write out a level-0 run
  - There are $\left\lceil \frac{B(R)}{M} \right\rceil$ level-0 sorted runs
- **Pass $i$**: merge $(M - 1)$ level-$(i - 1)$ runs at a time, and write out a level-$i$ run
  - $(M - 1)$ memory blocks for input, 1 to buffer output
  - # of level-$i$ runs = $\left\lceil \frac{\# \text{ of level-}(i-1) \text{ runs}}{M-1} \right\rceil$
- **Final pass** produces one sorted run
Performance of external merge sort

• Number of passes: \[ \left\lfloor \log_{M-1} \left[ \frac{B(R)}{M} \right] \right\rfloor + 1 \]

• I/O’s
  • Multiply by \( 2 \cdot B(R) \): each pass reads the entire relation once and writes it once
  • Subtract \( B(R) \) for the final pass
  • Roughly, this is \( O(B(R) \times \log_M B(R)) \)

• Memory requirement: \( M \) (as much as possible)

Recall: We do not count the final write!
Some tricks for sorting

• Double buffering
  • Allocate an additional block for each run
  • Overlap I/O with processing
  • Trade-off: smaller fan-in (more passes)

• Blocked I/O
  • Instead of reading/writing one disk block at time, read/write a bunch (“cluster”)
  • More sequential I/O’s
  • Trade-off: larger cluster → smaller fan-in (more passes)
Lecture 10c: Join Algorithms: Sort-Merge Joins
Notation

- Relations: $R, S$
- Tuples: $r, s$
- Number of tuples: $|R|, |S|$
- Number of disk blocks: $B(R), B(S)$
- Number of memory blocks available: $M$
- Cost metric
  - Number of I/O’s
  - Memory requirement

Remember our disk-memory diagram?
Sort-merge join

\[ R \bowtie_{R.A=S.B} S \]

- Sort \( R \) and \( S \) by their join attributes; then merge \( r, s \) = the first tuples in sorted \( R \) and \( S \)
  Repeat until one of \( R \) and \( S \) is exhausted:
    - If \( r.A > s.B \) then \( s = \) next tuple in \( S \)
    - else if \( r.A < s.B \) then \( r = \) next tuple in \( R \)
    - else output all matching tuples, and \( r, s = \) next in \( R \) and \( S \)

- I/O’s: \text{sorting} + 2B(R) + 2B(S) (always?)
  - In most cases (e.g., join of key and foreign key)
  - Worst case is \( B(R) \cdot B(S) \): everything joins
Example of merge join

\[ R: \]
\[ r_1.A = 1 \]
\[ r_2.A = 3 \]
\[ r_3.A = 3 \]
\[ r_4.A = 5 \]
\[ r_5.A = 7 \]
\[ r_6.A = 7 \]
\[ r_7.A = 8 \]

\[ S: \]
\[ s_1.B = 1 \]
\[ s_2.B = 2 \]
\[ s_3.B = 3 \]
\[ s_4.B = 3 \]
\[ s_5.B = 8 \]

\[ R \bowtie_{R.A=S.B} S: \]
\[ r_1 s_1 \]
\[ r_2 s_3 \]
\[ r_2 s_4 \]
\[ r_3 s_3 \]
\[ r_3 s_4 \]
\[ r_7 s_5 \]
Optimization of SMJ

- **Idea**: combine join with the (last) merge phase of merge sort
- **Sort**: produce sorted runs for $R$ and $S$ such that there are fewer than $M$ of them total
- **Merge and join**: merge the runs of $R$, merge the runs of $S$, and merge-join the result streams as they are generated!
Performance of SMJ

• If SMJ completes in two passes:
  • I/O’s: $3 \cdot (B(R) + B(S))$ - why 3?
  • Memory requirement
    • We must have enough memory to accommodate one block from each run: $M > \frac{B(R)}{M} + \frac{B(S)}{M}$
    • $M > \sqrt{B(R) + B(S)}$

• If SMJ cannot complete in two passes:
  • Repeatedly merge to reduce the number of runs as necessary before final merge and join
Other sort-based algorithms

• Union (set), difference, intersection
  • More or less like SMJ

• Duplication elimination
  • External merge sort
    • Eliminate duplicates in sort and merge

• Grouping and aggregation
  • External merge sort, by group-by columns
    • Trick: produce “partial” aggregate values in each run, and combine them during merge
      • This trick doesn’t always work though
        • Examples: SUM(DISTINCT ...), MEDIAN(...)
Lecture 10d:
Join Algorithms: Hash Joins
Notation

- Relations: $R, S$
- Tuples: $r, s$
- Number of tuples: $|R|, |S|$
- Number of disk blocks: $B(R), B(S)$
- Number of memory blocks available: $M$

Cost metric
  - Number of I/O’s
  - Memory requirement
Hash join

\( R \bowtie_{R.A=S.B} S \)

- **Main idea**
  - Partition \( R \) and \( S \) by hashing their join attributes, and then consider corresponding partitions of \( R \) and \( S \)
  - If \( r.A \) and \( s.B \) get hashed to different partitions, they don’t join

Nested-loop join considers all slots

Hash join considers only those along the diagonal!
Partitioning phase

- Partition $R$ and $S$ according to the same hash function on their join attributes

Same for $S$
Probing phase

- Read in each partition of $R$, stream in the corresponding partition of $S$, join
  - Typically build a hash table for the partition of $R$
    - Not the same hash function used for partition, of course!

![Diagram showing disk and memory partitions with load, stream, and join steps]
Performance of (two-pass) hash join

• If hash join completes in two passes:
  • I/O’s: \(3 \cdot (B(R) + B(S))\)
  • Memory requirement:
    • In the probing phase, we should have enough memory to fit one partition of \(R\): \(M - 1 > \frac{B(R)}{M-1}\)
    • \(M > \sqrt{B(R)} + 1\)
    • We can always pick \(R\) to be the smaller relation, so:
      \[M > \sqrt{\min(B(R), B(S))} + 1\]
Generalizing for larger inputs

• What if a partition is too large for memory?
  • Read it back in and partition it again!
    • See the duality in multi-pass merge sort here?
Other hash-based algorithms

• Union (set), difference, intersection
  • More or less like hash join

• Duplicate elimination
  • Check for duplicates within each partition/bucket

• Grouping and aggregation
  • Apply the hash functions to the group-by columns
  • Tuples in the same group must end up in the same partition/bucket
  • Keep a running aggregate value for each group
    • May not always work
Lecture 10d:

Join Algorithms: Index-nested Loop Joins (and other use of Index)
Notation

• Relations: $R, S$
• Tuples: $r, s$
• Number of tuples: $|R|, |S|$
• Number of disk blocks: $B(R), B(S)$
• Number of memory blocks available: $M$
• Cost metric
  • Number of I/O’s
  • Memory requirement
Selection using index

- Equality predicate: $\sigma_{A=v}(R)$
  - Use an ISAM, B⁺-tree, or hash index on $R(A)$
- Range predicate: $\sigma_{A>v}(R)$
  - Use an ordered index (e.g., ISAM or B⁺-tree) on $R(A)$
  - Hash index is not applicable

- Indexes other than those on $R(A)$ may be useful
  - Example: B⁺-tree index on $R(A, B)$
  - How about B⁺-tree index on $R(B, A)$?
Index versus table scan

Situations where index clearly wins:

- **Index-only queries** which do not require retrieving actual tuples
  - Example: $\pi_A(\sigma_{A>v}(R))$

- Primary index clustered according to search key
  - One lookup leads to all result tuples in their entirety
Index versus table scan (cont’d)

BUT(!):

• Consider $\sigma_{A>v}(R)$ and a secondary, non-clustered index on $R(A)$
  • Need to follow pointers to get the actual result tuples
  • Say that 20% of $R$ satisfies $A > v$
    • Could happen even for equality predicates
• I/O’s for index-based selection: lookup + 20% $|R|$
• I/O’s for scan-based selection: $B(R)$
• Table scan wins if a block contains more than 5 tuples!
Index nested-loop join

\( R \bowtie_{R.A=S.B} S \)

• Idea: use a value of \( R.A \) to probe the index on \( S(B) \)
• For each block of \( R \), and for each \( r \) in the block:
  Use the index on \( S(B) \) to retrieve \( s \) with \( s.B = r.A \)
  Output \( rs \)

• I/O’s: \( B(R) + |R| \cdot \text{(index lookup)} \)
  • Typically, the cost of an index lookup is 2-4 I/O’s
  • Beats other join methods if \( |R| \) is not too big
  • Better pick \( R \) to be the smaller relation

• Memory requirement: 3
Zig-zag join using ordered indexes

\[ R \bowtie_{R.A=S.B} S \]

- Idea: use the ordering provided by the indexes on \( R(A) \) and \( S(B) \) to eliminate the sorting step of sort-merge join
- Use the larger key to probe the other index
  - Possibly skipping many keys that don’t match
Lecture 10e:
Discussion on join algorithms
Hash join versus SMJ

(Assuming two-pass)

- I/O’s: same
- Memory requirement: hash join is lower
  \[ \sqrt{\min(B(R), B(S))} + 1 < \sqrt{B(R) + B(S)} \]
  - Hash join wins when two relations have very different sizes
- Other factors
  - Hash join performance depends on the quality of the hash
    - Might not get evenly sized buckets
  - SMJ can be adapted for inequality join predicates
  - SMJ wins if \( R \) and/or \( S \) are already sorted
  - SMJ wins if the result needs to be in sorted order
What about nested-loop join?

• May be best if many tuples join
  • Example: non-equality joins that are not very selective

• Necessary for black-box predicates
  • Example: WHERE user_defined_pred(R.A, S.B)
Duality of sort and hash

• Divide-and-conquer paradigm
  • Sorting: physical division, logical combination
  • Hashing: logical division, physical combination

• Handling very large inputs
  • Sorting: multi-level merge
  • Hashing: recursive partitioning

• I/O patterns
  • Sorting: sequential write, random read (merge)
  • Hashing: random write, sequential read (partition)
Summary of techniques

• Scan
  • Selection, duplicate-preserving projection, nested-loop join

• Sort
  • External merge sort, sort-merge join, union (set), difference, intersection, duplicate elimination, grouping and aggregation

• Hash
  • Hash join, union (set), difference, intersection, duplicate elimination, grouping and aggregation

• Index
  • Selection, index nested-loop join, zig-zag join