1. Risk attitudes. (We imagine that) COVID-19 has been eliminated from
the world and Bob is making plans for Spring Break. He most prefers to go
to Cancun, a trip that would cost him $3000. Another good option is to go to
Miami, which would cost him only $1000. Bob is really excited about Spring
Break and cares about nothing else in the world right now. As a result, Bob’s
utility \( u \) as a function of his budget \( b \) is given by:

\[
\begin{align*}
  & u(b) = 0 \text{ for } b < 1000, \\
  & u(b) = 1 \text{ for } 1000 \leq b < 3000, \\
  & u(b) = 2 \text{ for } b \geq 3000.
\end{align*}
\]

Bob’s budget right now is $1100 (which would give him a utility of 1, for going
to Miami).

Bob’s wealthy friend Alice is aware of Bob’s predicament and wants to offer
him a “fair gamble.” Define a fair gamble to be a random variable with expected
value $0. An example fair gamble (with two outcomes) is the following: $-75
with probability 2/5, and $50 with probability 3/5. Note that the expected value
of this gamble is $0, so it is indeed fair. If Bob were to accept this gamble, he
would end up with $1025 with probability 2/5, and with $1150 with probability
3/5. In either case, Bob’s utility is still 1, so Bob’s expected utility for accepting
this gamble is \((2/5) \cdot (1) + (3/5) \cdot (1) = 1\).
a (10 points). Find a fair gamble with two outcomes that would strictly decrease Bob’s expected utility.

b (10 points). Find a fair gamble with two outcomes that would strictly increase Bob’s expected utility.

2. Finding Nash equilibria of normal-form games. (50 points.)

Find all the Nash equilibria of each of the following five two-player normal-form games. Argue why the games have no other Nash equilibria. (Hint: for some of these games, you may wish to use strict dominance or iterated strict dominance, because any strategy eliminated by (iterated) strict dominance cannot get positive probability in any Nash equilibrium. Also keep in mind that you may want to use strict dominance by a mixed strategy.)

4, 4 8, 2
2, 8 7, 7
0, 8 4, 0
2, 0 0, 1
7, 7 6, 8
9, 2 0, 1
3, 5 5, 4
1, 7 7, 6

4, 0 4, 0 1, 2
3, 5 3, 4 2, 4
4, 0 1, 1 5, 0

3. Extensive-form games. Consider the game in Figure 1.

Figure 1: An extensive-form game with imperfect information.

a (10 points). Give the normal-form representation of this game.
b (10 points). Give a Nash equilibrium where player 1 sometimes plays Left. (Remember that you must specify each player’s strategy at every information set.)
c (10 points). What are the subgame perfect equilibria of the game? (Remember that you must specify each player’s strategy at every information set.)


This question is a programming question. Please see Homework 1 for details about getting set up with GLPK, making a directory for this homework, etc.

Note: in this question, there is an example instance that you are asked to solve. However, just getting this example right is not enough to get full credit: your formulation should work on all instances. The example is just there to give you something to test your formulation on.

Elena Umberta Massima is an expected utility maximizer. When presented with two probability distributions over a set of possible outcomes, E.U.M. says, without hesitation, which she prefers, and you will not catch her in any inconsistencies.

We have four outcomes: A, B, C, D. We will accordingly represent probability distributions as vectors of four probabilities of the respective outcomes. \((p_A, p_B, p_C, p_D) \succ (p'_A, p'_B, p'_C, p'_D)\) will denote that E.U.M. prefers distribution \(p\) to \(p'\). We learn the following four preferences:

- \((.1, .2, .3, .4) \succ (.1, .2, .4, .3)\)
- \((.4, .4, .1, .1) \succ (.4, .2, .2, .2)\)
- \((.6, .1, 0, .3) \succ (.4, .3, .3, 0)\)
- \((.4, .3, .2, .1) \succ (.5, .5, 0, 0)\)

Obviously, we jump on the opportunity to estimate the utilities (for outcomes A, B, C, D) of this fascinating woman.

Our goal will be to assign utilities in the interval \([0, 1]\) to the four outcomes that are consistent with E.U.M.’s preferences. Write a linear program formulation for this. You should add an objective to satisfy the consistency constraints by as large a margin as possible (similar to our linear program for strict dominance by mixed strategies). You should use the MathProg (.mod) language to model the general problem (you should allow for more than four outcomes and more than four preferences) and solve the specific instance above. (Hint: the optimal objective value is 0.02.)