Complementary slackness

- if \( a^T x \geq b \) is a primal constraint, and \( y \) is the corresponding dual variable, then for any pair of optimal solutions, \( y (a^T x - b) = 0 \)

- if \( a^T x > b \) (primal constraint is not tight) then
  \( y = 0 \) (dual variable = 0)

- if \( y > 0 \) (dual variable positive) then
  \( a^T x = b \) (primal constraint is tight)

Simplex algorithm

- Basic feasible solution
  - a basic feasible solution of a linear program with \( n \) variables is a feasible solution equal to the solution of a system of \( n \) linear equations where each equation is a tight constraint.

- Relies on convexity

A linear program has \( n \) variables \( m \) constraints \((m \geq n)\)

A basic feasible solution is a feasible solution where \( n \) out of \( m \) constraints are set to equalities.

Ellipsoid algorithm
- separation oracle: given a candidate solution (assignment of $x$)
decide $x$ is feasible or output a constraint which $x$ violates.

- LP for min spanning tree

  $x_{ij}$ variable for edge $ij$, $x_{ij} = \begin{cases} 1 & \text{if } (i,j) \text{ is in } \mathcal{A} \\ 0 & \text{otherwise} \end{cases}$

  $0 \leq x_{ij} \leq 1$

  for every cut $C$, $\sum_{(i,j) \in C} x_{ij} \geq 1$

- interior point

  - barrier function
    
    $x \geq 0 \Rightarrow \left\{ \begin{array}{ll}
    \frac{1}{x} & \mbox{if } x \neq 0 \\
    \frac{1}{e^x - 1} & \mbox{if } x = 0
    \end{array} \right.$

    $1 - x - y \geq 0 \Rightarrow \left\{ \begin{array}{ll}
    \frac{1}{1 - x - y} & \mbox{if } 1 - x - y > 0 \\
    \frac{1}{e^{1-x-y} - 1} & \mbox{if } 1 - x - y = 0
    \end{array} \right.$