- Counter examples for some ideas of interval scheduling

A. schedule the meeting that starts earliest

Example: (1, 10)  (2, 4)  (5, 7)

Rule A will choose (1, 10), but that is bad because the optimal solution has two meetings (2, 4) (5, 7)

C. schedule the meeting that takes least amount of time

Example:  (5, 7)  (2, 6)  (6, 10)

Rule C will choose (5, 7), but optimal solution (2, 4) (6, 10)

- Running the algorithm

  \[
  (1, 3), (2, 4), (4, 5), (1, 6), (6, 8)
  \]

  \[
  1 \ 3 \ 4 \ 5 \ 6 \ 8
  \]

- Proof of correctness for the interval scheduling

  Proof: assume towards contradiction that the algorithm is not optimal.

  Then there must be an instance of interval scheduling where the algorithm does not schedule max number of meetings.

  Let algorithm's solution for this instance be  \( A_{CL} = \{ i_1, i_2, \ldots, i_c \} \), where meetings are sorted by end-time.

  Let the optimal solution for this instance be  \( O = \{ i_1', i_2', \ldots, i_c' \} \) indices of the meetings scheduled.
let the optimal solution for this instance be
\[ \text{OPT} = (u_1, u_2, u_3, \ldots, u_{l'}) \]
\[ l' > l \]
\[ \text{OPT scheduled: } ALG \text{ scheduled.} \]

compare two solutions ALG and OPT, let \( k \) be the first index
where \( i_k \neq j_k \) (ALG and OPT disagrees with the choice
of \( k \)-th meeting).

Claim: the solution \((i_1, i_2, \ldots, i_k, j_{k+1}, j_{k+2}, \ldots, j_{l'})\) is also
an optimal solution that scheduled \( l' \) meetings.

proof of Claim: by design of the algorithm.

meeting \( i_k \) ends no later than meeting \( j_k \)
\[ \text{end} i_k \leq \text{end} j_k \leq \text{start} j_{k+1} \]
\[ \text{OPT is a valid solution} \]
\[ i_k \text{ does not conflict with } j_{k+1} \]

repeat the above argument until for all \( 1 \leq k \leq l \), \( i_k = j_k \)
that means \((i_1, i_2, \ldots, i_k, j_{k+1}, \ldots, j_{l'})\) is a valid solution
this cannot happen because \( j_{k+1} \) ends after \( i_k \), so it must
be considered by the algorithm after \( i_k \) is scheduled, by
design of algorithm, \( ALG \) would also include \( j_{k+1} \), contradiction
by proof of contradiction, the assumption is false, \( ALG \) always
finds the optimal solution

(more formal proof: prove by induction (on \( k \)) that
\[(i_1, i_2, \ldots, i_k, j_{k+1}, j_{k+2}, \ldots, j_{l'})\] is a valid solution