Compsci 201
Big-Oh, Interfaces, Maps

<table>
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<th>N</th>
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</tr>
</tbody>
</table>

Susan Rodger
February 7, 2020
I is for …

• Interface
  • LinkedList implements List

• Inheritance
  • LinkedList extends AbstractSequentialList
Announcements

• Assignment P2 out today, due Thur. Feb 13
  • Get it done early, great practice for exam
• APT-3 due yesterday
  • Last chance to turn in today til 11:59pm
• Discussion 5 on Feb 10
  • Prepare for exam
• Exam next week, Feb 14
PFOWBE

• Big-Oh and O-Notation
  • Building a mathematical formalism with intuition

• Interfaces: List, Set, and Map
  • When it makes sense to use general type
  • Empirical and Analytical measures of efficiency

• Maps: API and Problem Solving
  • Keys and Values
Review ListSplicer.java, removeFirst

- [https://coursework.cs.duke.edu/201spring20/classcode/](https://coursework.cs.duke.edu/201spring20/classcode/)
- Declarations – using List<> interface
  ```java
  List<String> linked = new LinkedList<>();
  List<String> array = new ArrayList<>();
  ```
- Method removeFirst, parameter list
  ```java
  public double removeFirst(List<String> list) {
  ```
- Method removeFirst pass either list
  ```java
  double ltime = splicer.removeFirst(linked);
  double atime = splicer.removeFirst(array);
  ```
list.remove(0)

- list is LinkedList or ArrayList, call List<> methods

```java
public double removeFirst(List<String> list) {
    double start = System.nanoTime();
    while (list.size() != 1) {
        list.remove(index: 0);
    }
    double end = System.nanoTime();
    return (end - start) / 1e9;
}
```

- If list is ArrayList – call remove for ArrayList
- If list is LinkedList, call remove for LinkedList
What is “faster”? LinkedList or ArrayList

\[ y = -4E-05x + 0.0009 \]

\[ y = 0.0064x^2 - 0.0156x + 0.0238 \]

R\(^2\) = 0.9984

<table>
<thead>
<tr>
<th>Data Points</th>
<th>Linear (linked)</th>
<th>Polynomial (array)</th>
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</thead>
<tbody>
<tr>
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<td>0.0118</td>
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<tr>
<td>1000000</td>
<td>0.0173</td>
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</tbody>
</table>
Access all elements randomly

- What is “faster”? **LinkedList** or **ArrayList**

![Random Access Chart]

- Random Access
  
  - y = 0.1292x^2 - 0.7137x + 1.3337
  - R² = 0.9889
  - y = 0.0002x + 5E-05
  - R² = 0.8169

<table>
<thead>
<tr>
<th>x</th>
<th>y values</th>
<th>R² values</th>
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</thead>
<tbody>
<tr>
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<td>8.0320</td>
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</tr>
<tr>
<td>20000</td>
<td>76.7703</td>
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<td>30000</td>
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<td>40000</td>
<td>204.9666</td>
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</table>
Measuring Efficiency

• Which is faster, LinkedList or ArrayList?
  • What does it depend on?
  • Empirically: depend on computer used?

• ArrayList.remove(0):
  • \( y = 0.0064x^2 - 0.0156x + 0.0238 \)
  • \( R^2 = 0.9984 \)
Random Access Efficiency

- Random Access for Lists
  - `alist.get(N/2)` versus `llist.get(N/2)`
  - Does this depend on N?

- LinkedList random access of n elements n times
  - \( y = 0.0129x^2 - 0.7137x + 1.3337 \)
  - \( R^2 = 0.9889 \)

- ArrayList random access of n elements n times
  - \( y = 0.0002x + 5E-05, R^2 = 0.8169 \)
Big-Oh aka O-Notation

• Intuition: behavior in the limit matters
  • What happens as $N$ gets large, where we measure performance in terms of $N$
  • For polynomials: leading term, no coefficients

\[ y = 3x \quad y = 6x - 2 \quad y = 15x + 44 \]
\[ y = x^2 \quad y = x^2 - 6x + 9 \quad y = 3x^2 + 4x \]

• The **first family is** $O(n)$, the **second is** $O(n^2)$
More on O-Notation

• **Provides theoretical analysis.** Independent of, and can obscure some, empirical details
  - Compare: 20N hours v N^2 microseconds
  - Which is better? Does it depend?

• If an algorithm is O(N) it’s also O(N^2) from a technical, mathematical perspective
  - O is an upper bound, in the limit
  - We try to provide tight, or best bounds/analysis
Binary search: guess number 1-1024, hi, lo, correct
   # of guesses? $O(\log N)$  note $2^{10} = 1024$
   If 12 seconds for $2^{10}$ then 24 seconds for $2^{20}$

Sequential/linear search: every element of list
   # elements examined? $O(N)$
   If 12 seconds for $2^{10}$ then 24 seconds for $2^{11}$
   Double input, double time
Big-Oh for More Algorithms

• Efficient sorting: merge, quick, Tim
  • # elements examined? $O(N \log N)$
  • More time than linear, but not terrible

• Looking at every pair, or slow sorting, e.g., bubble
  • # elements examined? $O(N^2)$
  • 12 seconds for $2^{10}$ then 144 seconds for $2^{11}$
  • Double the input, square the time
Running times in seconds
machine: $10^9$ instructions/sec

<table>
<thead>
<tr>
<th>$N$</th>
<th>$O(\log N)$</th>
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<th>$O(N \log N)$</th>
<th>$O(N^2)$</th>
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</tr>
<tr>
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<td>0.0199</td>
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</tr>
<tr>
<td>1,000,000,000</td>
<td>0.000000003</td>
<td>1.002</td>
<td>65.8</td>
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2/7/2020 CompSci 201, Spring 2020
Running times in seconds
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<td>65.8</td>
<td>'</td>
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<td>$1.002$</td>
<td>$65.8$</td>
<td>$31.8\text{ years}$</td>
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</table>
What do they all have in common?
What do they all have in common?

• They all took a CompSci course at Duke!
ArrayList<String> words = new ArrayList<>();
words.add("cat");
words.add("fish");
words.add("dog");
String b = words.get(1);
words.set(2, "frog");
int c = words.indexOf("cat");
words.set(1, words.get(c));
ArrayList Methods

```java
ArrayList<String> words = new ArrayList<>();
words.add("cat");
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String b = words.get(1);
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```

Words is [ ]
ArrayList Methods

ArrayList<String> words = new ArrayList<>();
words.add("cat");
words.add("fish");
words.add("dog");
String b = words.get(1);  
words.set(2, "frog");
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Words is [ cat ]
ArrayList Methods

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ArrayList<String> words = new ArrayList<>();
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```

Words is [ cat, fish]
ArrayList Methods

ArrayList<String> words = new ArrayList<>();
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Words is [ cat, fish, dog]
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int c = words.indexOf("cat");
words.set(1, words.get(c));

Words is [ cat, fish, frog]
b is fish c is 0
ArrayList Methods

ArrayList<String> words = new ArrayList<>();
words.add(“cat”);
words.add(“fish”);
words.add(“dog”);
String b = words.get(1);
words.set(2, “frog”);
int c = words.indexOf(“cat”);
words.set(1, words.get(c));

Words is [ cat, cat, frog]  
b is fish  
c is 0
Problems and Solutions

• String that occurs most in a list of strings?
  • CountingStringsBenchmark.java, two ideas
    • See also CountingStringsFile for same ideas
    • https://coursework.cs.duke.edu/201spring20/classcode
  • Parallel arrays: word[k] occurs count[k] times
  • Use ArrayLists: 2 “the”, 3 “fat”, 4 “fox”

<table>
<thead>
<tr>
<th>the</th>
<th>fox</th>
<th>cried</th>
<th>fat</th>
<th>tears</th>
</tr>
</thead>
<tbody>
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<tr>
<td>2</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
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<td>3</td>
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<td>1</td>
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<td>2</td>
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</tbody>
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How does the code work?

• Process each string $s$
  • First time $\text{words.add}(s), \text{counter.add}(1)$
  • Otherwise, increment count corresponding to $s$
  • $c[x] += 1$ ?

```java
public static String parallelArrays(List<String> list) {
    ArrayList<String> words = new ArrayList<>();
    ArrayList<Integer> counter = new ArrayList<>();

    for(String w : list) {
        int index = words.indexOf(w);
        if (index == -1){
            words.add(w);
            counter.add(1);
        } else {
            counter.set(index, counter.get(index) + 1);
        }
    }
    return words.toString();
}
```
What is complexity of this code?

- Search for each word and ... if occurs at k
  - +1 to counter.get(k), else add at end

- Search complexity? O(M) when M different words
  - One search is O(M) – what about all searches?
  - Tracking all words. First time zero, then one, ...
  - Avoid analyzing duplicates for the moment
    - Will take longer if we have multiple occurrences of some of M words
Tracking N strings

- Complexity of search? $O(M)$ for $M$ different words
  - Complexity of `words.indexOf(...)` is $O(M)$
  - what about all calls? $1 + 2 + \ldots + N$ is $N(N+1)/2$

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public static String parallelArrays(List<String> list) {
    ArrayList<String> words = new ArrayList<>();
    ArrayList<Integer> counter = new ArrayList<>();

    for (String w : list) {
        int index = words.indexOf(w);
        if (index == -1) {
            words.add(w);
            counter.add(1);
        } else {
            counter.set(index, counter.get(index) + 1);
        }
    }
}
```

$O(N^2)$
Should we be more precise?

• Adding M different words will be $O(M^2)$
  • $1 + 2 + \ldots + M = M(M+1)/2$

• Adding duplicates: we need to be precise about adding N total words.
  • Sometimes word will be found, still $O(M)$ for M different words
  • We have both M and N here, but treat M == N for easier analysis.
CountingStringsFile.java

• Generate an ArrayList of Strings
  • Find the word that occurs the most often
    • See three different methods
Understanding O-notation

• This is an upper bound and in the limit
  • Coefficients don’t matter, order of growth
  • $N + N + N + N = 4N$ is $O(N)$ --- why?
  • $N^2N$ is $O(N^2)$ – why?
  • $O(1)$ means independent of $N$, constant time

• In analyzing code and code fragments
  • Account for each statement
  • How many times is each statement executed?
Why coefficients don’t matter

<table>
<thead>
<tr>
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<th>100N</th>
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<tbody>
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Why coefficients don’t matter

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<td>1,000,000,000,000</td>
<td>20,000,000,000,000</td>
</tr>
<tr>
<td>10,000,000,000</td>
<td>(20)\times10,000,000,000</td>
<td>(100)\times10,000,000,000</td>
<td>(2000)\times10,000,000,000</td>
</tr>
</tbody>
</table>
Just Say No.. When you can

\[ O(n^2) \]
Example: Analyze using big-Oh

- What is runtime of `stuff(N)`
  - How to reason about this
- What is return value of `stuff(N)`
  - What if code changes to `sum += k`

```java
public int stuff(int n) {
    int sum = 0;
    for(int k=0; k < n; k += 1) {
        sum += n;
    }
    return sum;
}
```
Counting for O-notation

• Why is O(1) complexity of sum += n
  • Is this O(1) for any x += y?
  • Loop executes N times, doing O(1) per iteration
    • Total runtime for method? O(n)

```java
public int stuff(int n) {
    int sum = 0;
    for(int k=0; k < n; k += 1) {
        sum += n;
    } 
    return sum; 
}
```
Example 2: Analyzing O-Notation

• What is big-Oh of runtime of call \texttt{calc(N)}?  
  • Num. of statements executed, \( O(1) \) for line 146?  
  • Use \texttt{calc(16)} and generalize

• What is big-Oh of value returned by \texttt{calc(N)}?  
  • Table? \( k = 1, 2, 4, 8, 16, 32, 64 \)