Experimenting with Grammars to Generate L-Systems – in JFLAP
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L-Systems

- Grammatical systems introduced by Lyndenmayer
- Model biological systems and create fractals
- Similar to Chomsky grammars, except all variables are replaced in each step, not just one!
- Successive strings are interpreted as strings of render commands and displayed graphically
Parts of an L-System  
(a type of grammar)

• Defined over an alphabet

• Three parts
  – Axiom (starting place)
  – Replacement rules (replaces all variables at once)
  – Geometric rules (for drawing)
    • g means move forward one unit with pen down
    • f means move forward one unit with pen up
    • + means turn right by the default angle
    • - means turn left by the default angle
L-System

An L-system is composed of three parts \((\Sigma, h, w)\)

- \(\Sigma\): finite alphabet set of symbols
- \(h\): rewriting rules each symbol is replaced by string of symbols
- \(w\): axiom starting point

\(h\) is finite substitutions, \(h:\Sigma \to \Sigma^*\).
\( h(w) \)

\( h(w) \) is computed by replacing every symbol in \( w \) that has a rewrite rule by that rule.

A language \( L \) of an L-system is the word sequence generated by

\[
\begin{align*}
\bullet & \ h^0(w) = w \\
\bullet & \ h^1(w) = h(w) \\
\bullet & \ h^2(w) = h(h(w)) \\
\bullet & \ \ldots \\
L & = \{ h^i(w) \mid i \geq 0 \}
\end{align*}
\]
NOTE: If \( h(a) = bb \) we will write this as a rule

\[ a \rightarrow bb \]
Example:

\[ \Sigma \text{ alphabet: } \{a, b\} \]

\[ h \text{ rules: } \begin{align*}
    a & \rightarrow \text{ aa} \\
    b & \rightarrow \text{ ab}
\end{align*} \]

\[ w \text{ axiom: } \text{ ab} \]

What is the language \( L \) of strings represented by this \( L \)-system?

\[ L = \]
Drawing a picture of an L-system

Defining an L-system: (3 parts in this order)

- Axiom definition: This must be the first line of the file
- Production rules: Defines the replacement rules.
- Geometric rules: Defines colors, widths, etc.
Graphically represent

Symbols for drawing and moving:

- g: draw a line one step in the current direction
- f: move forward one step in the current direction
Example: example1

axiom X

X -> g f g X

distance 15
linewidth 5
color black

L =
What does this draw?
Geometric rules

• +    change direction to the right
• −    change direction to the left
• %    change direction 180 degrees
• ~    decrement the width of the next lines
• [    save in stack current state info
• ]    recover from stack state info
• {    start filled in polygon
• }    end filled in polygon
Example – lsys-samp1

- Axiom
- Replacement Rules
- Geometric Rules

NOTE: Must use spaces as separator between symbols
Example – lsys-samp1(cont)

• Derivation of strings

\[ \text{Note: replace both X and Y each time} \]
Example – lsys-samp2
Example – lsys-samp2 (cont)

\[
g[\sim+Yg]gX
\]

\[
g[\sim+++Yg]gg[\sim+Yg]gX
\]

\[
g[\sim++++Yg]gg[\sim+++Yg]gg[\sim+Yg]gX
\]

...
Example - tree
Example – tree rendered
Stochastic Tree

• Add a rule $T \rightarrow T$

• Now there is a choice for $T$, draw a line or don’t
Same Stochastic L-System

- Rendered 3 times, each at 8\textsuperscript{th} derivation
• JFLAP is available for free:
  www.jflap.org
• Duke School of Environment uses L-systems to model pine needles in Duke Forest