Section: Turing Machines - Building Blocks

1. Given Turing Machines M1 and M2

Notation for

- Run M1
- Run M2

M1

\[ \rightarrow S \rightarrow H \rightarrow \]  

M2

\[ \rightarrow S' \rightarrow H' \rightarrow \]  

\[ \rightarrow M1 \rightarrow M2 \rightarrow \]  

\[ \rightarrow S \rightarrow H \rightarrow z;z,R \rightarrow z;z,L \rightarrow S' \rightarrow H' \rightarrow \]  

z represents any symbol in
2. Given Turing Machines M1 and M2

\[ M_1 \xrightarrow{x} M_2 \]

\[ S \quad H \quad \xrightarrow{x;x,R} \quad z ; z , L \quad \xrightarrow{} \quad S' \quad H' \]

z represents any symbol in
x is an element of
3. Given Turing Machines M1, M2, and M3

M1

\[ \text{S} \quad \text{H} \]

\[ x \mapsto M2 \]

M2

\[ \text{S'} \quad \text{H'} \]

\[ x \mapsto M3 \]

M3

\[ \text{S''} \quad \text{H''} \]

x is an element of S H H'S'

y is any element except x from z

z is any element from x; x, R

z; z, L

y; y, R

z; z, L

S'' H''
More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

- $z$ is any symbol in $\Gamma$
- $x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right

3. $L$ - move left

4. $x$ - write $x$ (and don’t move)

5. $R_a$ - move right until you see an $a$
6. $L_a$ - move left until you see an $a$

7. $R_{\neg a}$ - move right until you see anything that is not an $a$

8. $L_{\neg a}$ - move left until you see anything that is not an $a$

9. $h$ - halt in a final state

10. $\begin{array}{l}
\begin{array}{l}
\begin{array}{l}
a, b \\
\end{array}
\end{array}
\end{array} \rightarrow \rightarrow \\
$If the current symbol is $a$ or $b$, let $w$ represent the current symbol.
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$.

If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb
input: ba, output: ba

What is the running time?
Example

Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}$, $|w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: $abbabb$, output: $abbabbbb$

The tape head should finish pointing at the leftmost symbol of $w$. 

Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An *algorithm* for a function $f: D \rightarrow R$ is a TM $M$, which given input $d \in D$, halts with answer $f(d) \in R$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

start with: $111 + 1111$

$\uparrow$

end with: $1111111$

$\uparrow$
Example: Copy a String, \( f(w) = w0w \), \( w \in \Sigma^* \), \( \Sigma = \{a, b, c\} \)

Denoted by \( C \)

\[
\begin{array}{ll}
\text{start with:} & \text{abac} \\
\uparrow & \\
\text{end with:} & \text{abac0abac} \\
\uparrow & \\
\end{array}
\]

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

\[
\begin{align*}
\text{start with:} & \quad \text{aaBbabca} \\
& \uparrow \\
\text{end with:} & \quad \text{aaBBbaca} \\
& \uparrow
\end{align*}
\]
Algorithm:

\begin{itemize}
\item remember symbol to the right and erase it
\item for each symbol to the left do
  \begin{itemize}
  \item shift the symbol one cell to the right
  \end{itemize}
\item replace first symbol erased
\item move tape head to appropriate position
\end{itemize}
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

\[
\begin{align*}
\text{start with:} & \quad \text{babcaBba} \\
\text{end with:}  & \quad \text{bacaBBBba}
\end{align*}
\]

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, \( f(x \times y) = x \times y \), \( x \) and \( y \) unary numbers. Assume \( x, y > 0 \).

\[
\begin{align*}
\text{start with:} & \quad 1111 \times 11 \\
\text{end with:} & \quad 11111111 \\
\end{align*}
\]