Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:
Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms

Definition of TM

- Storage
  - tape
- actions
  - write symbol
  - read symbol
  - move left (L) or right (R)
- computation
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

Formal Definition of TM

A TM $M$ is defined by $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ where

- $Q$ is finite set of states
- $\Sigma$ is input alphabet
- $\Gamma$ is tape alphabet
- $B \in \Gamma$ is blank
- $q_0$ is start state
- $F$ is set of final states
- $\delta$ is transition function

$\delta(q,a) = (p,b,R)$ means “if in state $q$ with the tape head pointing to an 'a', then move into state $p$, write a 'b' on the tape and move to the right”.
**TM as Language recognizer**

**Definition:** Configuration is denoted by $\vdash$.

If $\delta(q,a) = (p,b,R)$ then a move is denoted

$$abaqabba \vdash ababpbba$$

**Definition:** Let $M$ be a TM, $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$. $L(M) = \{ w \in \Sigma^* | q_0w \vdash^{*} x_1q_fx_2 \text{ for some } q_f \in F, x_1, x_2 \in \Gamma^* \}$

**TM as language acceptor**

$M$ is a TM, $w$ is in $\Sigma^*$,

- if $w \in L(M)$ then $M$ halts in final state
- if $w \notin L(M)$ then either
  - $M$ halts in non-final state
  - $M$ doesn’t halt

**Example**

$\Sigma = \{a, b\}$

Replace every second ‘a’ by a ‘b’ if string is even length.

- Algorithm
Example:

$L = \{a^n b^n c^n | n \geq 1\}$

Is the following TM Correct?

```
2;2,R
a;a,R

b;2,R

3;3,R
b;b,R

2;2,R
3;3,L
1;1,R

a;1,R

2;2,R
```

**TM as a transducer**

TM can implement a function: $f(w) = w'$

```
start with: w
↑

end with: w'
↑
```
**Definition:** A function with domain D is *Turing-computable or computable* if there exists TM 
\( M = (Q, \Sigma, \Gamma, \delta, q_0, B, F) \) such that

\[
q_0 w \xrightarrow{\ast} q_f f(w)
\]

for all \( q_f \in F \), for all \( w \in D \).

**Example:**

\( f(x) = 2x \)

\( x \) is a unary number

\[
\begin{array}{c}
\text{start with:} \\
111 \\
\uparrow \\
\text{end with:} \\
111111 \\
\uparrow \\
\end{array}
\]

Is the following TM correct?
Example:

$L = \{ww \mid w \in \Sigma^+\}, \Sigma = \{a, b\}$