Question 1

a. Given the following knowledge base:

\[ \forall x \ [\text{Knows}(Bill, x) \implies \text{Loves}(Bill, x)] \]
\[ \forall x, y \ [\text{Friend}(x, y) \implies \text{Friend}(y, x)] \]
\[ \forall x, y \ [\text{Friend}(x, y) \implies \text{Knows}(x, y)] \]
\[ \text{Friend}(Ted, Bill) \]

Prove:

\[ \text{Loves}(Bill, Ted) \]
b. Given the following knowledge base:

\[ \forall x \ [Knows(Bill, x) \implies Loves(Bill, x)] \]
\[ \forall x, y \ [Friend(x, y) \implies Friend(y, x)] \]
\[ \forall x, y \ [Friend(x, y) \implies Knows(x, y)] \]
\[ \exists x \ Friend(x, Bill) \]

Prove:

\[ \exists x \ Loves(Bill, x) \]
c. Given the following knowledge base:

\[ \forall x \ [\text{Knows}(Bill, x) \implies \text{Loves}(Bill, x)] \]
\[ \forall x, y \ [\text{Friend}(x, y) \implies \text{Friend}(y, x)] \]
\[ \forall x, y \ [\text{Friend}(x, y) \implies \text{Knows}(x, y)] \]
\[ \forall x \ [\exists y \ \text{Friend}(x, y)] \]

Prove:

\[ \exists x \ \text{Loves}(Bill, x) \]
Question 2

Let $A$, $B$ and $C$ be three discrete random variables with joint probability distribution $P(\cdot)$ which is a function of three variables.

**Notation:** Capital letters denote random variables, small letters are numbers that denote possible realizations of the random variables (i.e. values it can take). The letter $P$ is overloaded notation and means different things depending on the arguments passed. So $P(ABC)$ is the joint probability over $A$, $B$, $C$ and is a function that takes three arguments. Similarly, $P(BC)$ is the joint probability over $B$, $C$ and is a function that takes two arguments and $P(A)$ is the marginal probability of $A$, which is a function of a single variable. Also, $P(abc)$, $P(bc)$ and $P(a)$ are numbers that denote the probability of the events $(A = a \land B = b \land C = c)$, $(B = b \land C = c)$ and $(A = a)$ respectively.

Write the following in terms of the joint probability distribution.

a. $P(BC)$

b. $P(A)$

c. $P(A \lor B)$