Why "back of the envelope" estimates?

- Often need to make rapid estimates
  - to eliminate candidate solutions
  - establish feasibility
  - sketch out potential trade-offs
- Most remember key numbers related to their field, not every detail
- Hence we need to estimate
  - which numbers are important
  - values of numbers needed
  - how to perform the calculation
- Emphasis is on "order of magnitude" estimates
  - to nearest factor of 10 (or 2)

Orders of Magnitude

- How far away is home? Is it more like 1, or 10, or 100 miles?
  - Probably do not know exactly
  - Is it approximately "a couple", or "a few", or "a lot"
  - Estimate based on powers rather than multiples of 10
- How tall is your dorm? More like 1, 10, 100, 1000 feet?
  - 1 foot tall is like a doll house, so that's out
  - What do we know that is about 10 feet big? Hmm... People
  - If building is a couple of people high, 10 sounds good.
  - But that means 1000, would be 100 people high, so that's out
  - So 10 or 100 depending on how many people tall the building is
- Use orders of magnitude as brackets to find reasonable range

Example: How many piano tuners in NYC

- Approximately how many people are in New York City?
  - 10,000,000
- Does every individual own a piano?
  - No
- Reasonable to assert "individuals do not own pianos; families do"?
  - Yes
- About how many families are there in a city of 10 million people?
  - Perhaps there are 2,000,000 families
- Does every family own a piano?
  - No
- Perhaps one out of every five does
  - That would mean there are about 400,000 pianos in NYC

Example: Piano Tuners continued

- How many piano tuners are needed for 400,000 pianos?
  - Some people never get around to tuning their piano
  - Some people tune their piano every month
  - Assume "on the average" every piano gets tuned once a year, then there are 400,000 every year
- How many piano tunings can one piano tuner do?
  - Assume that average piano tuner can tune four pianos a day
  - Assume that there are 200 working days per year
  - That means every tuner can tune about 800 pianos per year
- How many piano tuners are needed in NYC?
  - Number of tuners is approximately 400,000/800 or 500

Example: Piano Tuners summary

- "Back of the Envelope" estimates have
  - Formulas: provide roadmap to upcoming calculations
  - Estimates: brief justification of approximations in formula
  - Calculations: estimates and known facts are use in formula
- Piano Tuner example
  - Formula:
    # tuners = # pianos x # repairs / # repairs per day x # days
  - Estimates
    # pianos ~= 400,000 (20% of 2,000,000 families own pianos)
    # repairs ~= 1 per piano (some many, some none)
    # repairs per day ~= 4
    #working days ~= 200 (5 x 50 – vacation, sickness)
  - Calculation
    # tuners ~= (400,000 x 1) / (4 x 200) = 500

Estimation General Principles

- Recall Einstein's famous advice
  - Everything should be made as simple as possible, but no simpler
- Do not worry about constant factors of 2, π, etc.
  - Round to "easy" number or nearest order of magnitude
- Guess numbers you do not know
  - Within bounds of common sense (accuracy increases with experience)
- Adjust geometry, etc., to suit you
  - Assume a cow is spherical if it helps
- Extrapolate from what you do know
  - Use ratios to assume unknown value is similar to known quantity
- Apply a 'plausibility' filter
  - If answer seems unbelievable, it probably is
  - Can usually set range of reasonable values that indicates major mistake (e.g., speed cannot be faster than light!)