CPS 296.2: Computational Game Theory and Mechanism Design. Midterm.

Your name:

Please read instructions carefully, and circle your final answers everywhere.
Try to do as much work as you can, but do not panic if you cannot solve
everything, I am fairly confident that nobody will. That is OK. Good luck!
–Vince

Question 1: True or false. (20 points.)
Label each statement as true or false. No explanation is required.

1. Converting a Bayesian game to normal form can cause an exponential blowup in space requirements.

2. Converting an extensive-form game to normal form can cause an exponential blowup in space requirements.

3. Every subgame-perfect equilibrium is also a Nash equilibrium.

4. If an extensive-form game does not have perfect information, i.e. there is at least one information set that consists of multiple nodes, then the extensive form gives us no useful strategic information and we may as well simply consider the normal form of the game.

5. If preferences are unrestricted and there are at least three alternatives, no reasonable deterministic voting rule is strategy-proof.

6. The Clarke mechanism is not vulnerable to collusion.

7. First-price sealed-bid auctions and Dutch auctions are equivalent from a strategic viewpoint.

8. The revenue equivalence theorem holds even when bidders are risk-averse.

9. With a single seller and a single buyer, each with privately held valuations for the item, there exists a strategy-proof, individually rational mechanism that allocates the item to the agent who values it more, and that only specifies payments that go from the buyer to the seller.

10. If telling the truth is a dominant strategy in a single-stage mechanism, then any elicitation protocol for that mechanism results in a multistage mechanism in which telling the truth is a dominant strategy.

11. (Bonus question - sorry, I couldn't resist.) Everyone taking this exam for credit will give this statement the same label (i.e. either everyone will label it "true" or everyone will label it "false").

Question 2: Finding Nash equilibria of normal-form games. (20 points.)

Find all Nash equilibria (both in pure and mixed strategies) of the following games. You do not need to prove that you have found all equilibria, or even to prove that they are equilibria, just list as many Nash equilibria as you can.

1. (Hint: think about strict dominance by mixed strategies.)

	L	M	R
U	6, 6	10, 7	3, 9
M	7, 10	8, 11	1, 10
D	9, 3	7, 1	0, 0

2. (Hint: think about weak dominance. Remember that weakly dominated strategies can sometimes occur in Nash equilibria, so reason carefully.)

	L	M	R
U	2, 2	0, 1	0, 5
D	1, 0	1, 1	1, 0

Question 3: Which voting rules choose the median voter's peak? (20 points.)

As we saw in class, when preferences are single-peaked (and there is an odd number of voters), the median voter's most preferred candidate (aka. "peak") is the Condorcet winner. But not every voting rule will actually select this candidate as the winner. Assume there are only three candidates (ordered on a line as a, b, c) and an odd number of single-peaked votes (i.e. nobody votes $a \succ c \succ b$ or $c \succ a \succ b$, i.e. b is never ranked last). For each of the following rules, say whether it will *always* select the median voter's most preferred candidate as the winner. If so, give a (hopefully one-line) explanation why. If not, give some single-peaked votes (an odd number) for which the rule will not select the median voter's most preferred candidate. Try to avoid situations with ties.

For example, for plurality (winner is candidate ranked first most often) the answer is "no": for 4 votes $a \succ b \succ c$, 3 votes $b \succ c \succ a$, and 2 votes $c \succ b \succ a$, the median voter votes for b, but a wins under plurality.

Now please answer the question for the following rules:

1. Copeland (winner is candidate with greatest number of pairwise wins)

2. Borda (2 points for being ranked first, 1 for being ranked second, 0 for being ranked third)

3. Maximin (winner is candidate that does best in its worst pairwise election)

4. Veto (1 point for being ranked first or second, 0 for being ranked third)

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Question 4: Finding an equilibrium of the "1 1/2th price sealed-bid auction." (20 points.)

In this question, we will analyze what we will call the "1 1/2th price sealed-bid auction." In this auction, the highest bidder wins and pays the *average* of the highest bid (i.e. her own bid) and the second-highest bid. Another way to interpret this (given that we assume risk-neutrality) is that after the auction is over, the auctioneer flips a coin to decide whether the winner should pay her own bid, or the second-highest bid.

Assume each bidder (n bidders total) draws a valuation independently from $[0, 1]$ (no interdependent valuations). Assume that preferences are quasilinear, i.e. the utility for i of winning is $v_i - p$ where p is the price the bidder pays (risk-neutral bidders). We will show that there exists a $k \in [0, 1]$ so that each bidder i bidding kv_i is a Bayes-Nash equilibrium, through the following steps.

1. Given that everyone else plays according to these strategies, what is the probability of winning if you bid $b \in [0, k]$?

2. Given that everyone else plays according to these strategies, and given that you win with a given bid $b \in [0, k]$, what is the probability that everyone else bids lower than $b' < b$? (Note that the probability density function of one other bidder's bid conditional on you winning with b is still uniform, but over a different range.)

3. Take the derivative of this expression with respect to b' to get the probability density function for the second-highest bid, conditional on you winning with bid b.

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4. Now, find the expected value of the second highest bid, conditional on you winning with bid b.

5. So, what is your expected utility as a function of your value v_i, conditional on you winning with bid b?

6. Now, what is your expected utility for bidding b (not conditional on you winning)? (Remember the first part of this question.)

7. Take the derivative of this expression, and set it equal to zero. Solve for b as a function of v_i. What is k? (Does your answer make sense intuitively? Remember that for the first-price auction, $k = (n-1)/n$, and for the second-price auction, $k = 1$.)

Question 5: A combinatorial auction for edges in a graph. (20 points.)

Suppose we are selling edges in an undirected graph (perhaps representing network capacity). Each bidder i is interested in obtaining a set of edges that constitute a path from some source node s_i to some target node t_i. Any path will give the bidder the same value v_i. Receiving more than one path from s_i to t_i is worthless (i.e. it will still give the bidder a total value of only v_i). Thus, a bid takes the form (s_i, t_i, v_i).
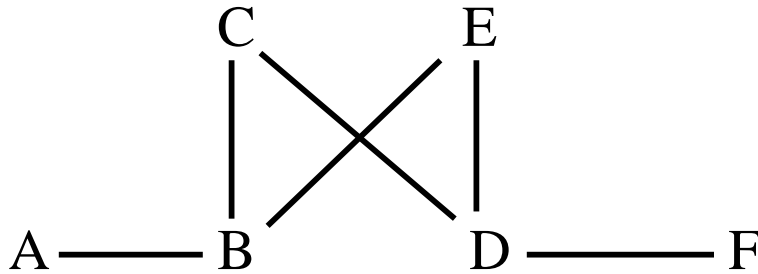
For example, consider the following graph.

Figure 1: A graph.

Suppose we receive the following bids:

Bidder 1: $(A, E, 4)$

Bidder 2: $(C, F, 2)$

Bidder 3: $(B, E, 1)$

Then, the optimal allocation is to give edges AB and BE to bidder 1, and edges CD and DF to bidder 2, for a total value of $4 + 2 = 6$. Note that it is impossible to accept bidder 3's bid in addition: there are only two paths from B to E, namely the one consising of edge BE, and the one consisting of edges BC, CD, and DE. Since edges BE and CD have been allocated already, neither of these paths can be allocated to bidder 3.

1. Compute the VCG (Clarke) payments for bidders 1 and 2.

2. Express each bidder's bid using the XOR language, i.e. as an XOR over bundles of edges (with values for each bundle).

3. In *general graphs*, is it possible that you will need an exponential number of XORs to express one of these source-target valuations? Justify your answer with an example or a proof.

4. What if you are allowed both ORs and XORs? Is it still possible that you will need an exponential number? Justify your answer with an example or a proof.