CPS 296.2 - Computational Game Theory and Mechanism Design

Homework 1 (due 9/14)

Note the rules for assignments on the course web page. Show all your work, but circle your final answer. Contact Vince (conitzer@cs.duke.edu) with any questions.

1. (Risk attitudes.) Bob is making plans for Spring Break. He most prefers to go to Cancun, a trip that would cost him \$2000. Another good option is to go to Miami, which would cost him only \$1000. Bob is really excited about Spring Break and cares about nothing else in the world right now. As a result, Bob's utility u as a function of his budget b is given by:

- $u(b) = 0$ for $b < \$1000$;
- $u(b) = 1$ for $\$1000 \leq b < \$2000$;
- $u(b) = 2$ for $b \geq \$2000$.

Bob's budget right now is \$1500 (which would give him a utility of 1, for going to Miami).

Bob's wealthy friend Alice is aware of Bob's predicament and wants to offer him a "fair gamble." Define a *fair gamble* to be a random variable with expected value \$0. An example fair gamble (with two outcomes) is the following: \$-150 with probability 2/5, and \$100 with probability 3/5. If Bob were to accept this gamble, he would end up with \$1350 with probability 2/5, and with \$1600 with probability 3/5. In either case, Bob's utility is still 1, so Bob's expected utility for accepting this gamble is $(2/5) \cdot (1) + (3/5) \cdot (1) = 1$.

a (5 points). Find a fair gamble with two outcomes that would strictly increase Bob's expected utility.

b (5 points). Find a fair gamble with two outcomes that would strictly decrease Bob's expected utility.

2. (Normal-form games.)

a (15 points). The following game has a unique Nash equilibrium. Find it, and prove that it is unique. (Hint: look for strict dominance.)

3, 1	1, 2	4, 0
0, 4	0, 4	3, 5
1, 2	2, 1	4, 0

b (15 points). Construct a single 2×2 normal-form game that simultaneously has all four of the following properties.

1. The game is not solvable by weak dominance (at least one player does not have a weakly dominant strategy).
2. The game is solvable by iterated weak dominance (so that one pure strategy per player remains).
3. In addition to the iterated weak dominance solution (which is a Nash equilibrium), there is a second pure-strategy Nash equilibrium.
4. Both players strictly prefer the second equilibrium to the first.

(Hints: the second pure-strategy equilibrium should not be strict; the pure-strategy equilibria should be in opposite corners of the matrix.) If you cannot get all four properties, construct an example with as many of the properties as you can.

c (15 points). Consider the following game:

3, 3	1, 4
4, 1	0, 0

Find a correlated equilibrium that places positive probability on all entries of the matrix, except the lower-right hand entry. Try to maximize the probability in the upper-left hand entry.

3. (Extensive-form games.) Consider the game below.

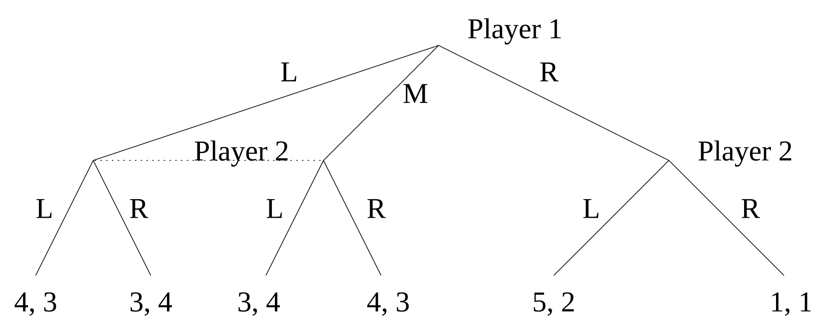

Figure 1: An extensive-form game with imperfect information.

a (15 points). Give the normal-form representation of this game.

b (15 points). Give a Nash equilibrium where player 1 sometimes plays left. (Remember that you must specify each player's strategy at *every* information set.)

c (15 points). Characterize the subgame perfect equilibria of the game. (Remember that you must specify each player's strategy at *every* information set.)