CPS 296.2 - Computational Game Theory and Mechanism Design

Homework 3 (due 11/21)

Note the rules for assignments on the course web page. Show all your work, but circle your final answer. Contact Vince (conitzer@cs.duke.edu) with any questions.

1. (Stochastic games.)

Consider the following two-state zero-sum stochastic game with discount factor .9 (i.e. you care about the next period only 90% as much as you care about the current period). (Each of the matrices corresponds to a state.)

30, -30 .4	0, 0 .4
0, 0 .4	20, -20 .4

-1, 1 .8	-1, 1 0
-2, 2 .9	0, 0 0

Each entry of each matrix gives the utilities for the players, as well as the probability of transitioning to the *other* state given that this entry was played. For example, if in state 1, (U, L) is played, then the next period the game will be in state 2 with probability .4 (and in state 1 with probability .6).

(30 points.) Find an equilibrium of this game in stationary strategies. You should give each player's strategy for each state, as well as the value of each state to player 1. You are allowed to solve this game by computer (e.g. by coding up Shapley's algorithm), but it should be possible to solve it by hand, if you think about the game carefully.

2. (Coalitional games.)

Consider again the archaic Babylonian Talmud game from class. Again, the wives have claims of 100, 200, and 300, respectively. Now suppose that the estate that the husband leaves behind is 250.

2a. (10 points.) Write down the characteristic function of the game. That is, for every nonempty subset of wives, write down the value of that subset. Remember that each coalition can choose to get 0, or to pay off everyone outside the coalition in full and divide the remainder of the estate. (Be careful – the rest of the question depends on this!)

2b. (10 points.) How much does the Shapley value give to each wife?

2c. (10 points.) Is the Shapley value outcome in the core? Why?

2d. (10 points.) How much does the nucleolus give to each wife? Write down the list of excesses for the nucleolus in decreasing order, and argue why it is impossible to better. (This question will probably require you to play around with the payoffs a little to find the nucleolus.)

2e. (0 points.) Do you believe that the Talmud would have recommended the nucleolus for this example? Why (not)?

3. (Using false names in a combinatorial auction.)

Consider a combinatorial auction with free disposal that uses the GVA mechanism. In this auction, all but one of the bidders are well-behaved—they only use one account. The false-name bidder can use an unlimited number of false names, and knows the bids of the well-behaved bidders.

(30 points.) Prove the following result: the false-name bidder can obtain all items for free if and only if no item receives a (positive) singleton bid from a well-behaved bidder. Be precise and rigorous.