

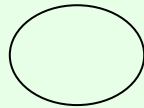
CPS 196.2

# Expressive negotiation over donations

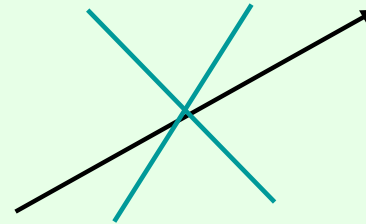
Vincent Conitzer  
[conitzer@cs.duke.edu](mailto:conitzer@cs.duke.edu)

# One donor (bidder)

$$u(\text{man}, \$100) = 1$$
$$u(\text{unicef}, \$100) = .8$$



$$U = 1$$

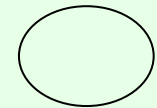
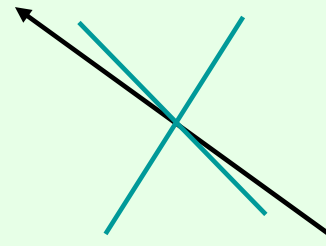
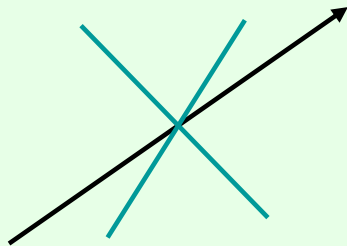
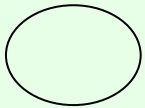


# Two independent donors

$u(\text{Man}, \$1) = 1$   
 $u(\text{UNICEF}, \$1) = .8$



$u(\text{Woman}, \$1) = 1$   
 $u(\text{UNICEF}, \$1) = .8$



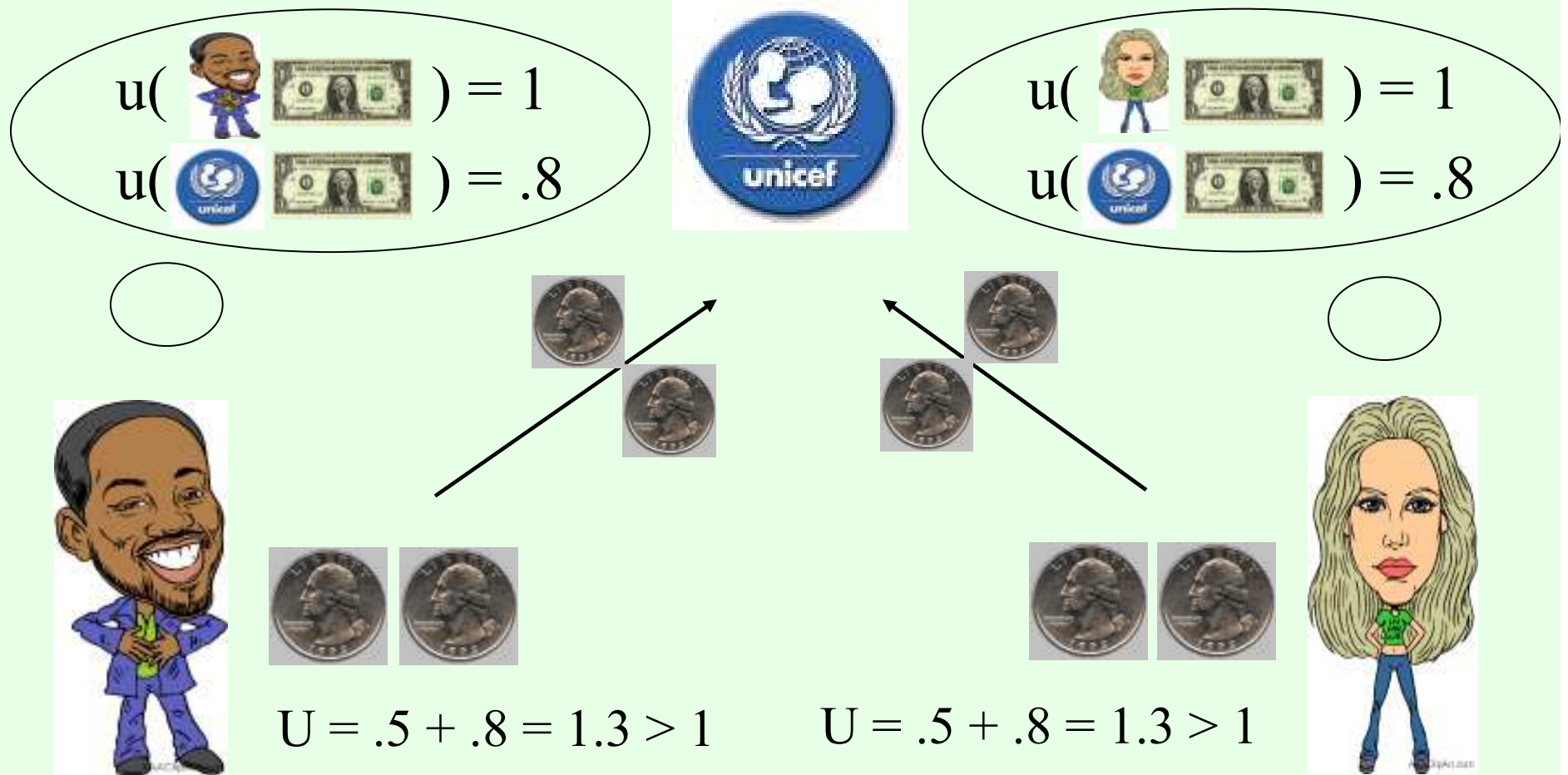
$U = 1$



$U = 1$



# Two donors with a contract



# Contracting using matching offers

I'll match any donation to



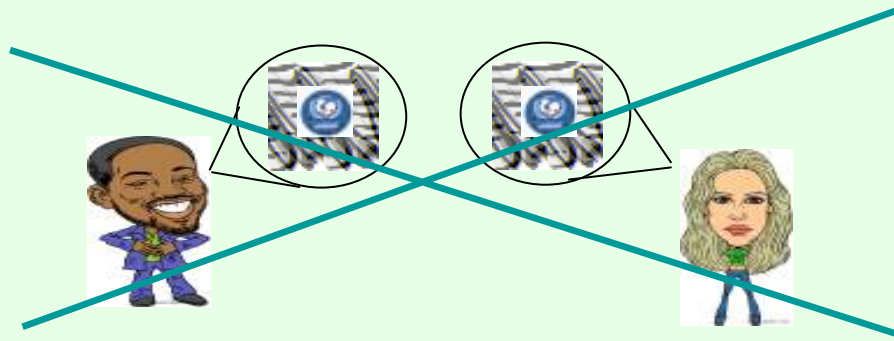
$$u(\text{woman}, \text{1 dollar}) = 1$$

$$u(\text{UNICEF logo}, \text{1 dollar and 2 quarters}) = 1.3$$

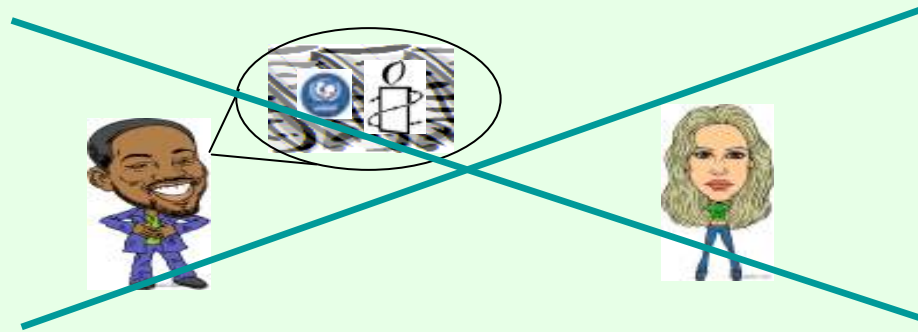


# Limitations of matching offers

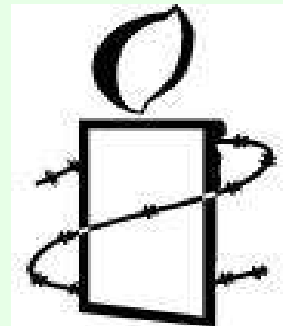
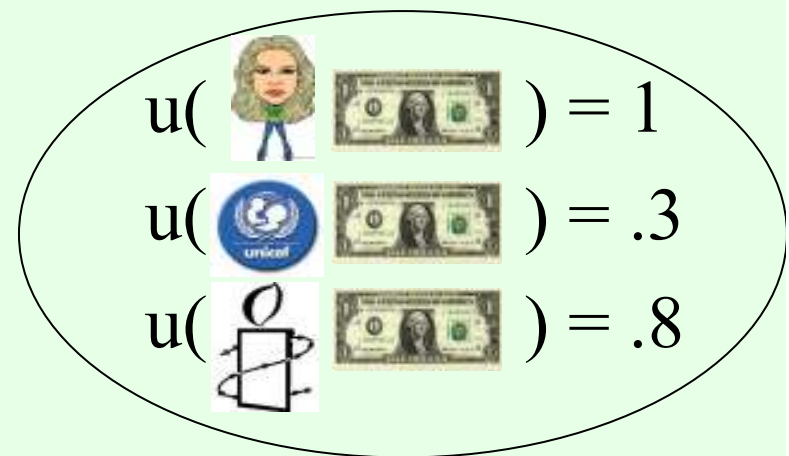
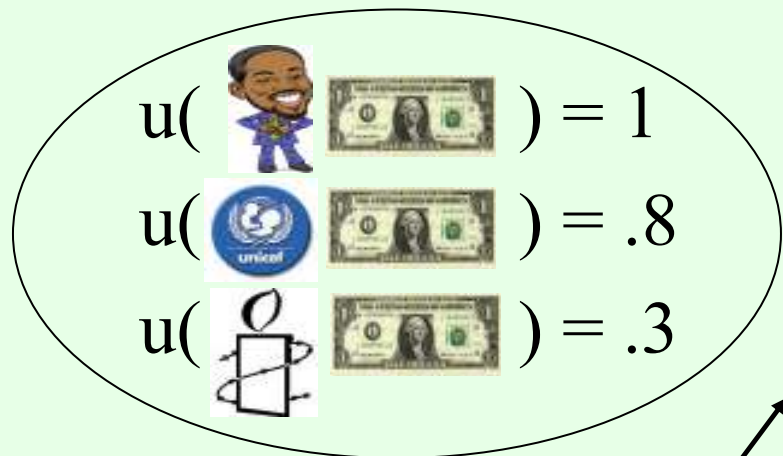
- One-sided



- Involve only a single charity



# Two charities



$U = 1.1$



$U = 1.1$



# A different approach

- Donors can submit **bids** indicating their preferences over charities
- A **center** accepts all the bids and decides who pays what to whom

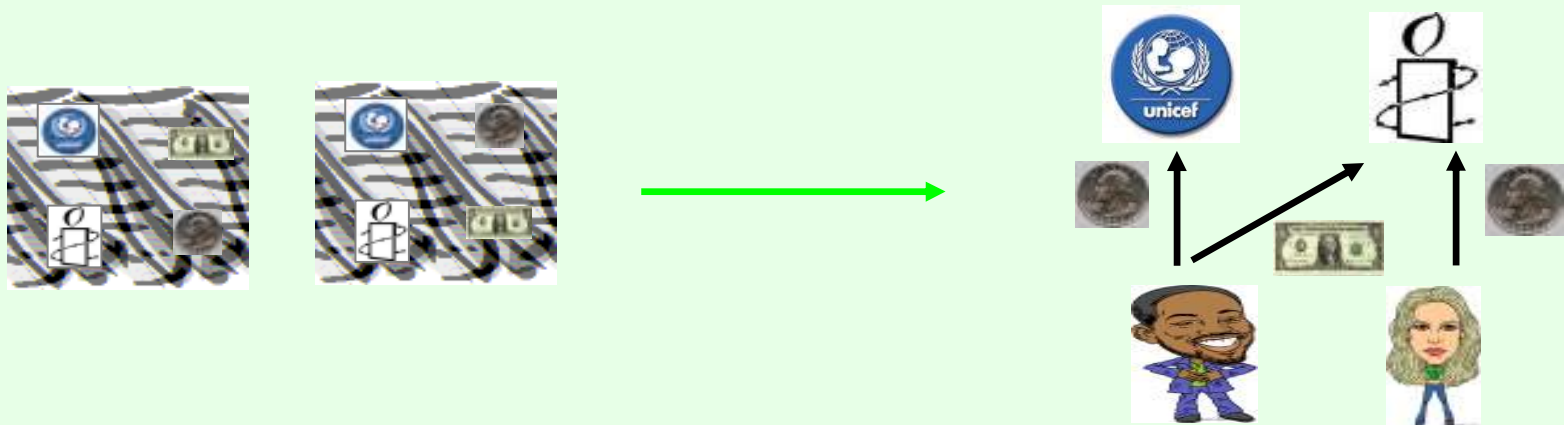


# What do we need?

- A general **bidding language** for specifying “complex matching offers” (*bids*)



- Algorithms for the **clearing problem** (given the bids, who pays what to whom)

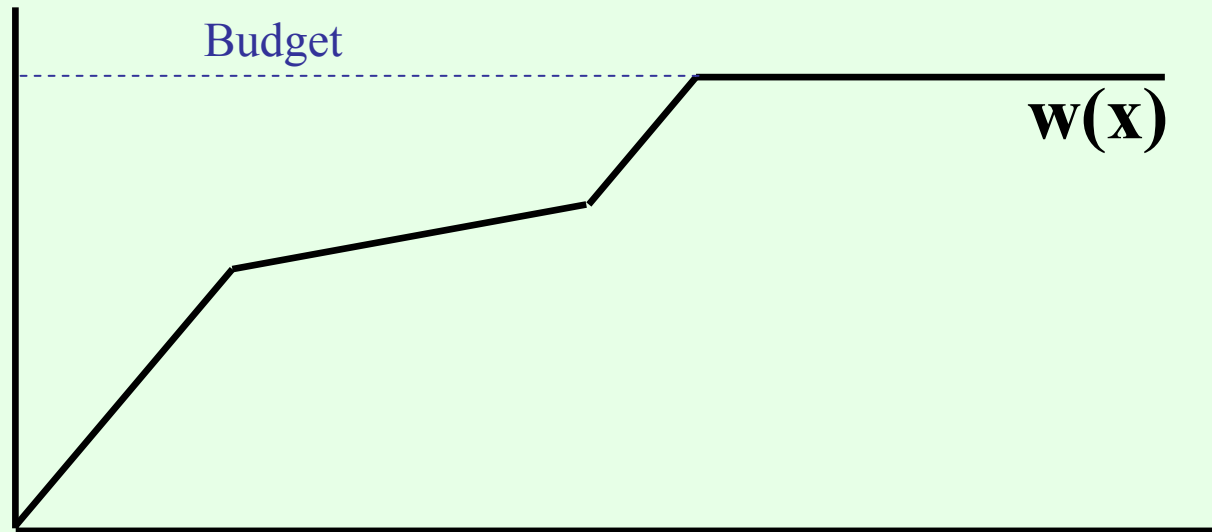


# One charity

- A bid for one charity:

*“Given that the charity ends up receiving a total of  $x$  (including my contribution), I am willing to contribute at most  $w(x)$ ”*

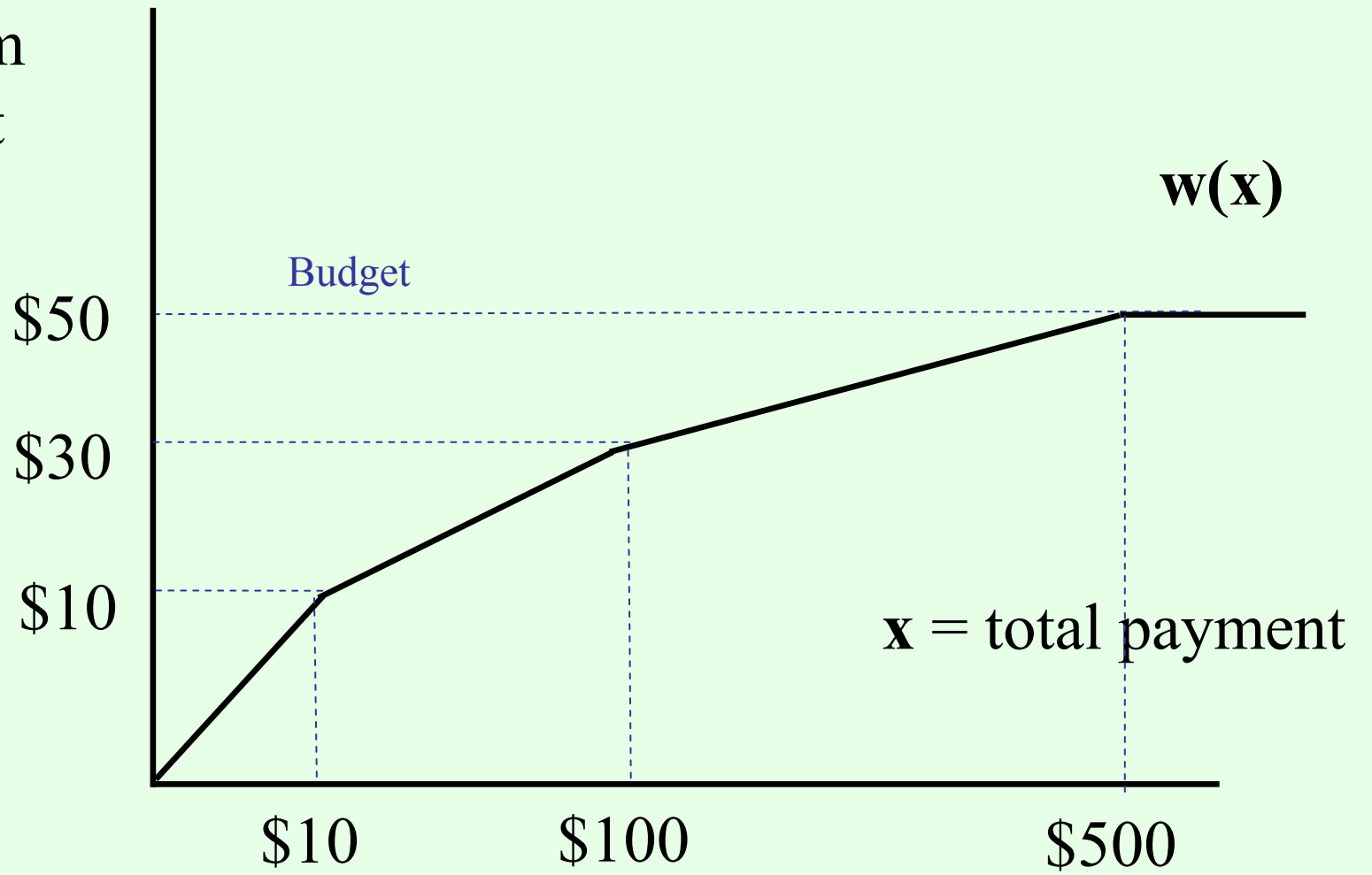
Bidder's  
maximum  
payment



$x$  = total payment to charity

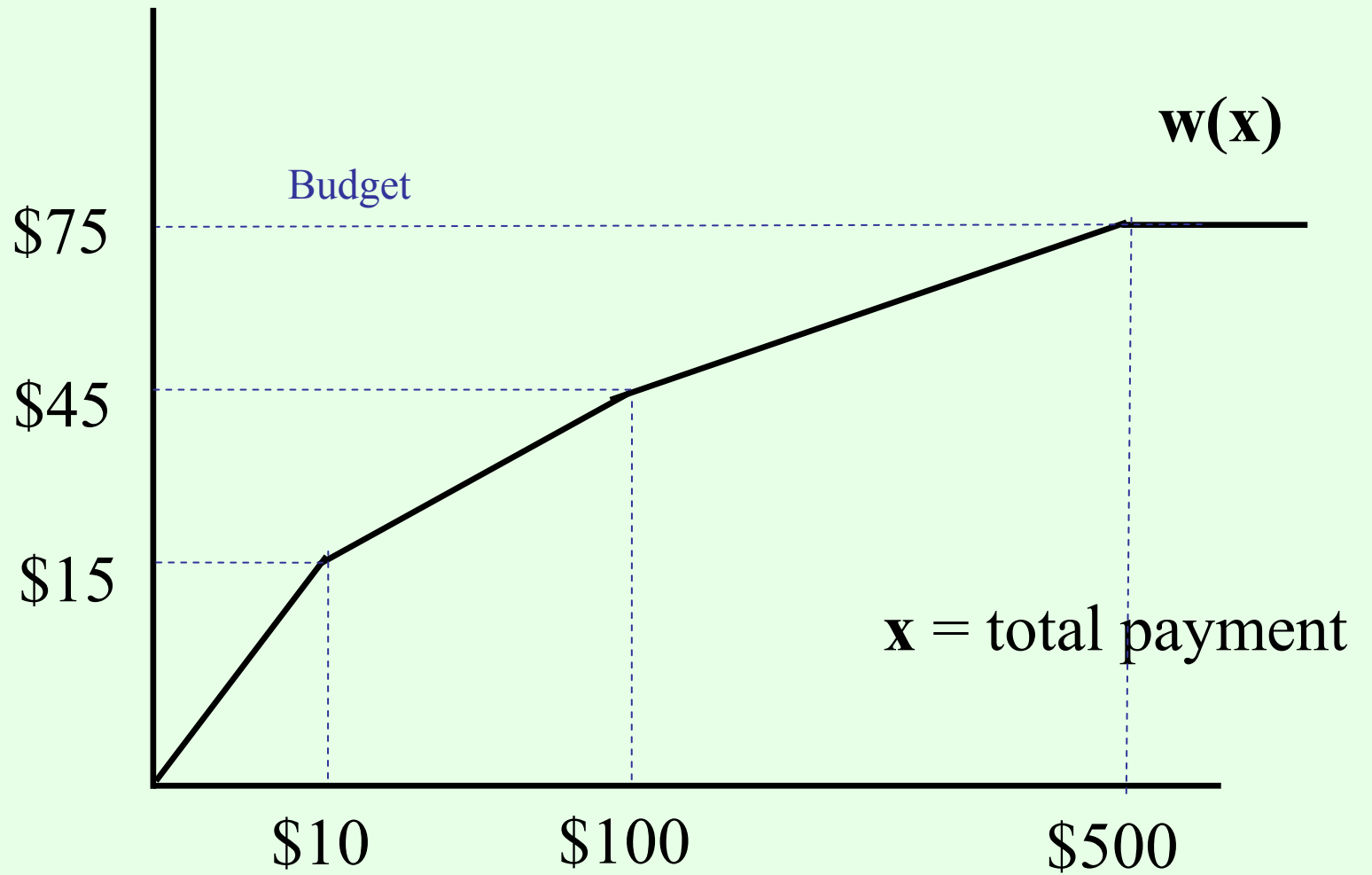
# Bid 1

maximum  
payment

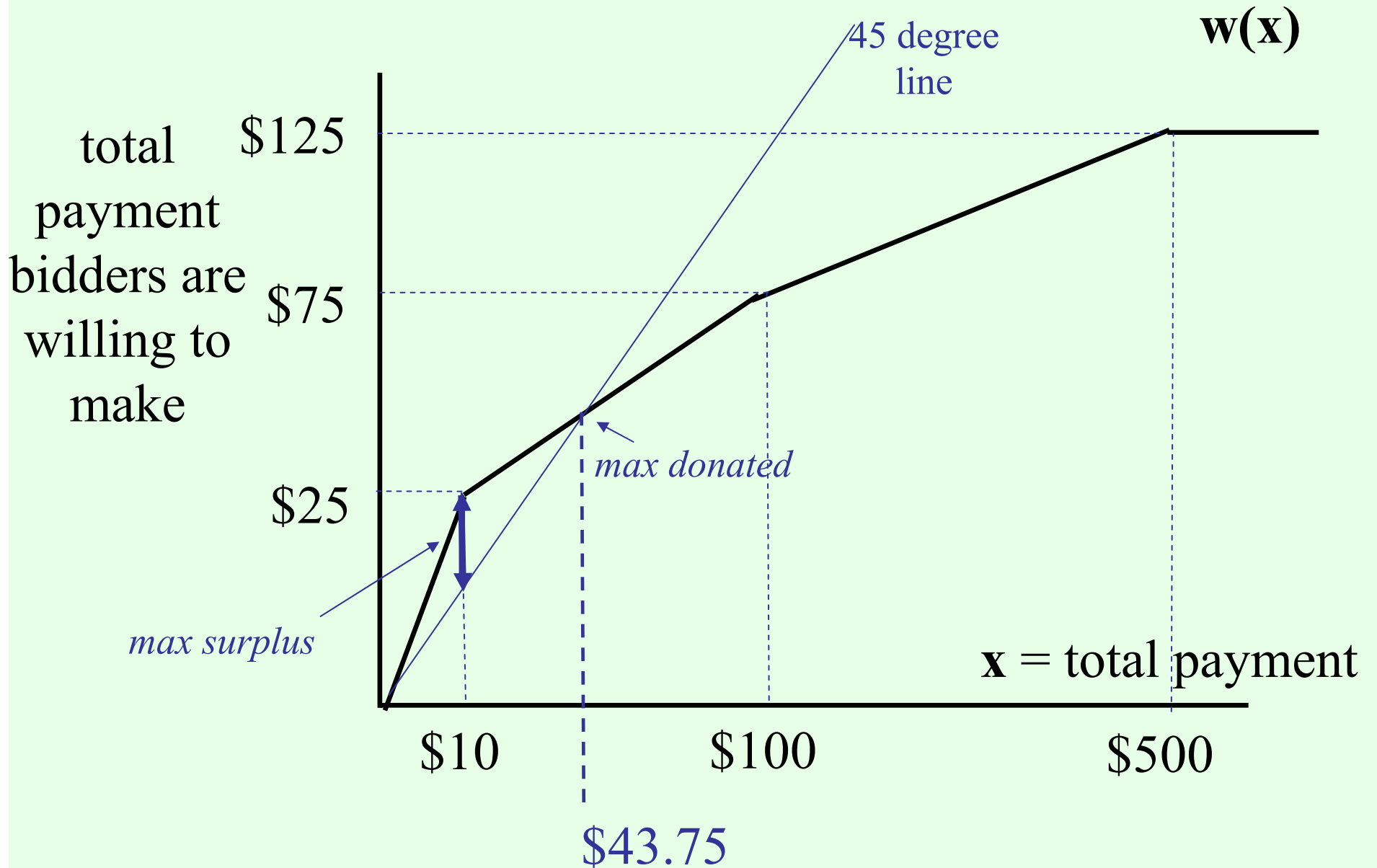


# Bid 2

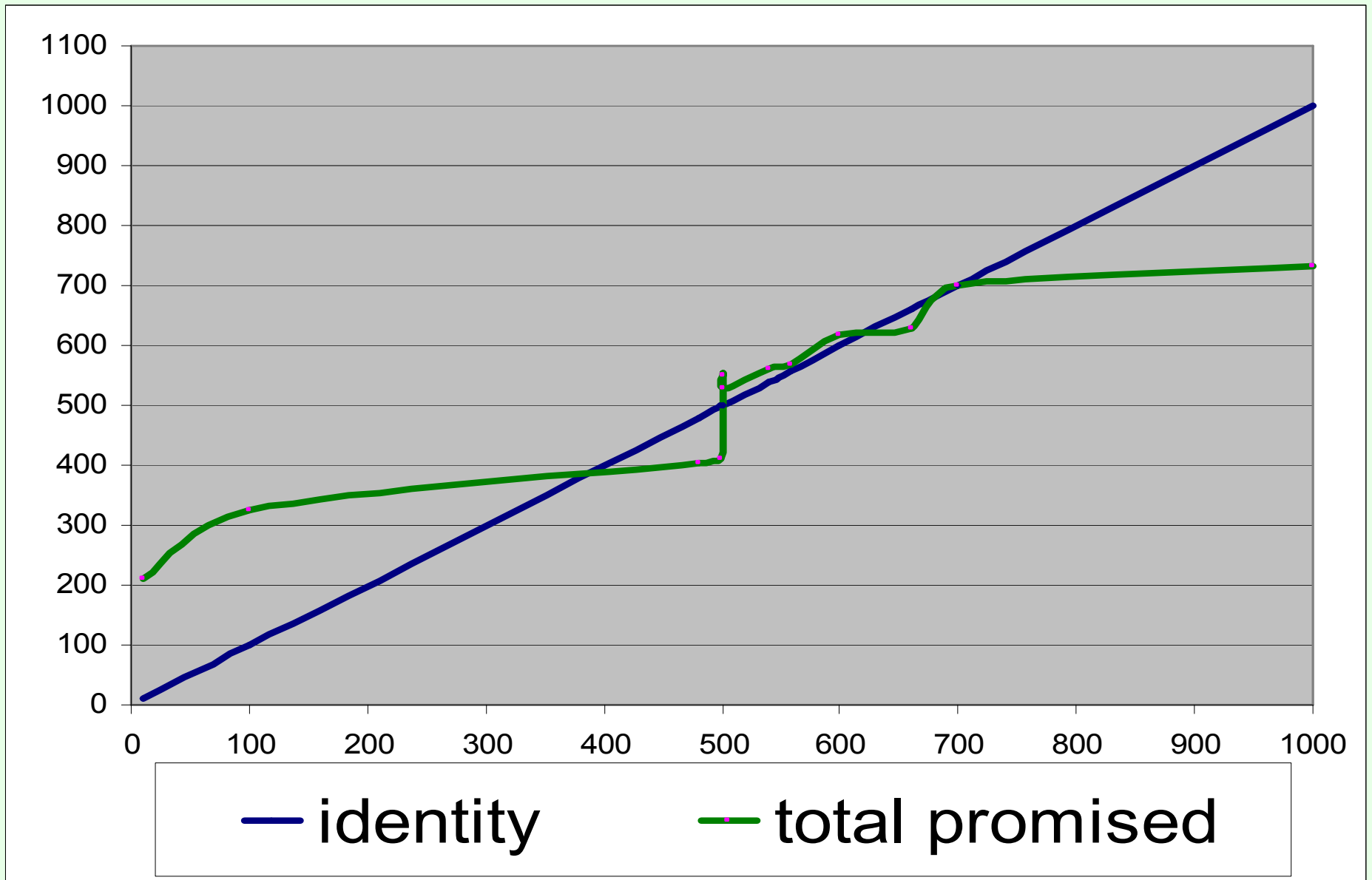
maximum  
payment



# Current solution

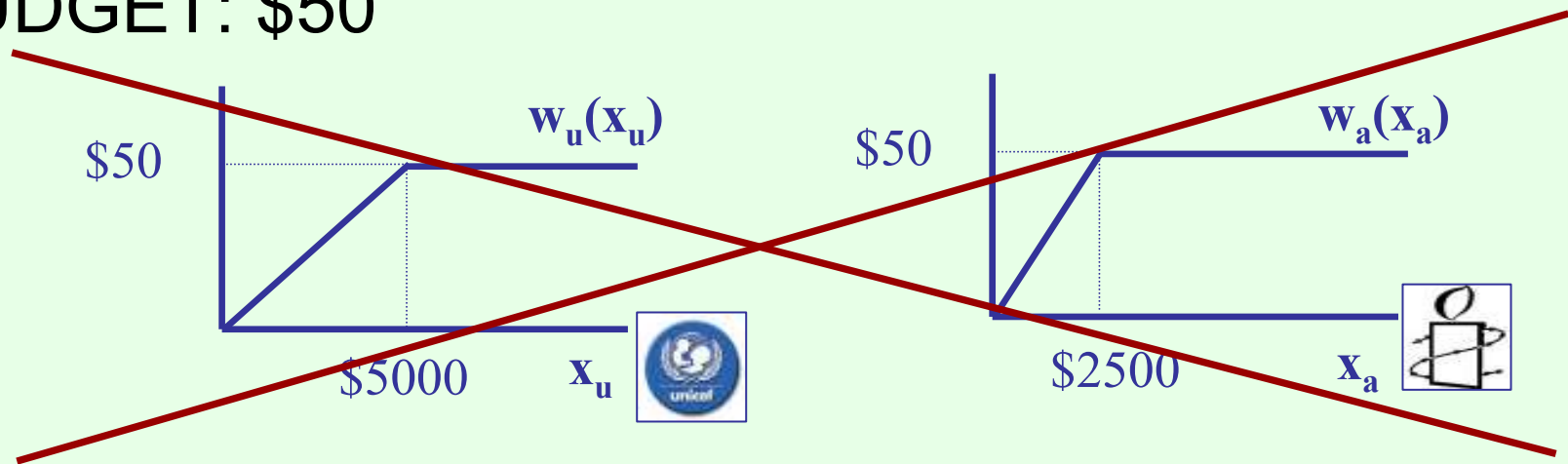


# Tsunami event (Dagstuhl 05)



# Problem with more than one charity

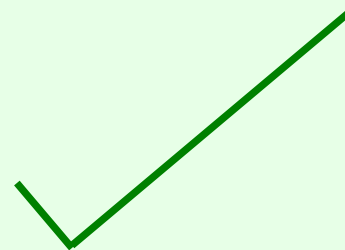
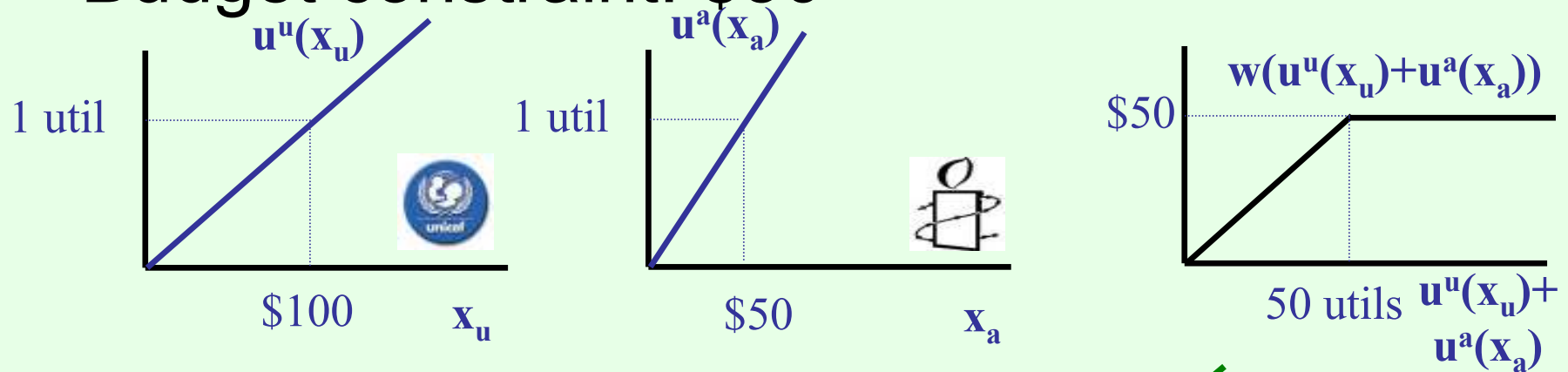
- Willing to give \$1 for every \$100 to UNICEF
- Willing to give \$2 for every \$100 to Amnesty Int'l
- BUDGET: \$50



- Could get stuck paying \$100!
- Most general solution:  $\mathbf{w}(x_1, x_2, \dots, x_m)$ 
  - Requires specifying exponentially many values

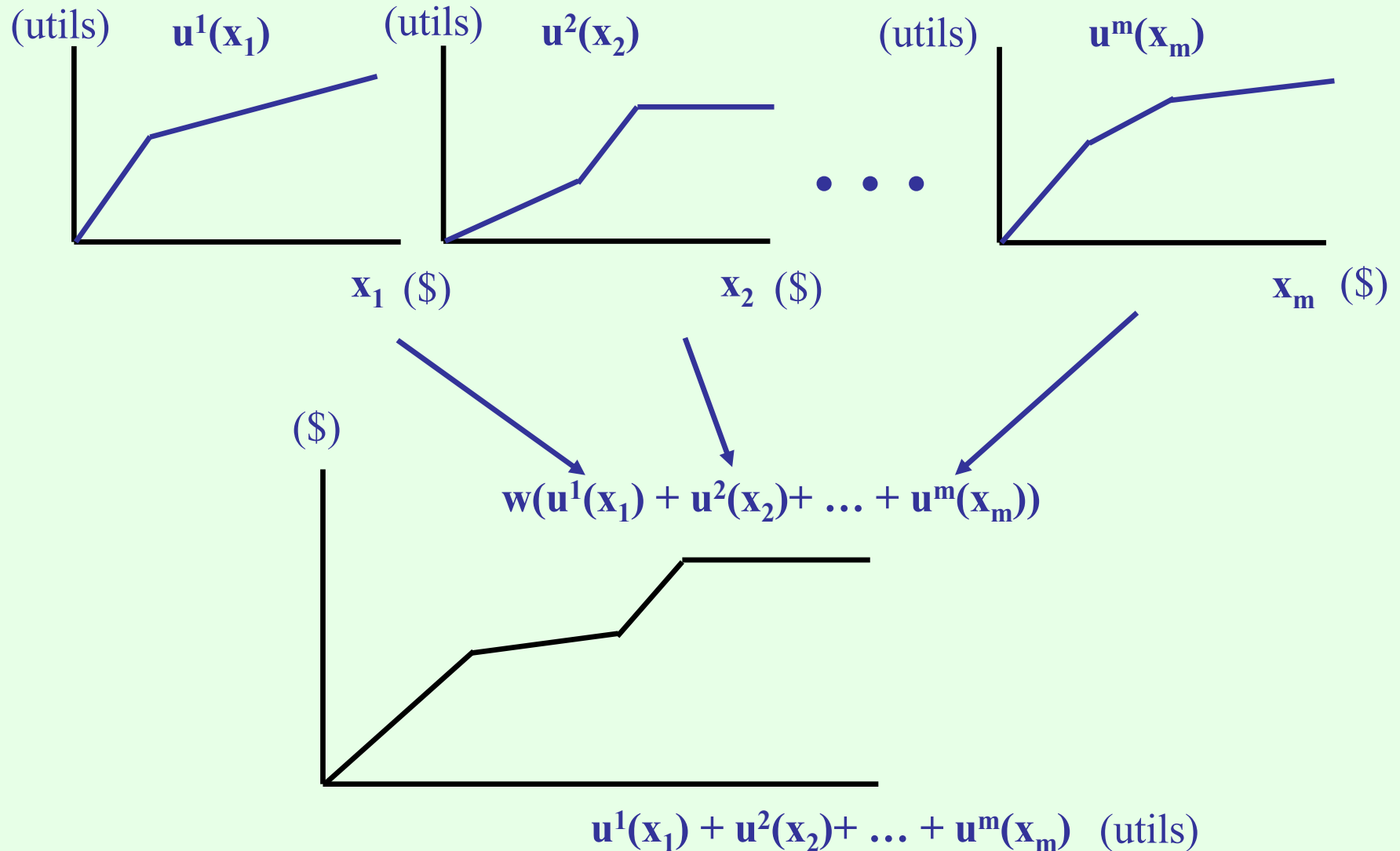
# Solution: separate *utility* and *payment*; assume utility decomposes

- Willing to give \$1 for every \$100 to UNICEF
- Willing to give \$2 for every \$100 to Amnesty Int'l
- Budget constraint: \$50



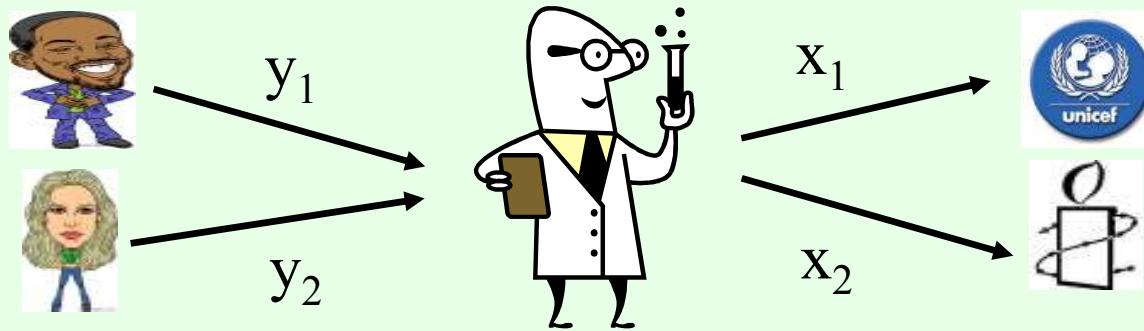


# The general form of a bid



# What to do with the bids?

- Decide  $x_1, x_2, \dots, x_m$  (total payment to each charity)
- Decide  $y_1, y_2, \dots, y_n$  (total payment by each bidder)

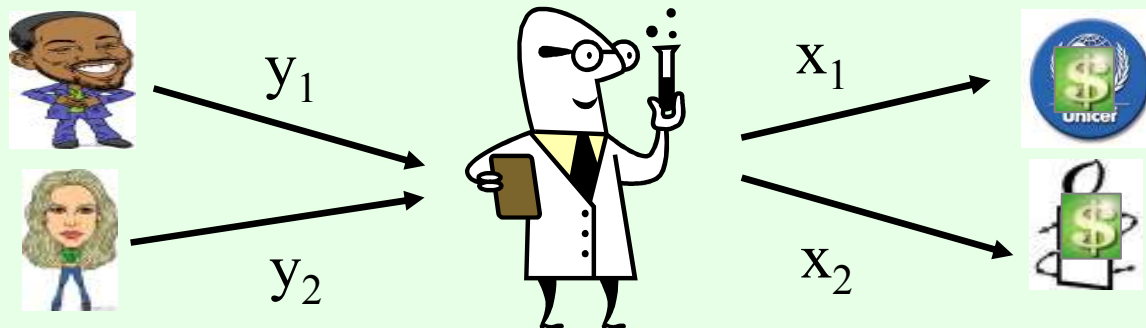


- Say  $x_1, x_2, \dots, x_m; y_1, y_2, \dots, y_n$  is *valid* if
  - $x_1 + x_2 + \dots + x_m \leq y_1 + y_2 + \dots + y_n$   
(no more money given away than collected)
  - For any bidder  $j$ ,  $y_j \leq w_j(u_j^1(x_1) + u_j^2(x_2) + \dots + u_j^m(x_m))$  (nobody pays more than they wanted to)

# Objective

- Among valid outcomes, find one that maximizes

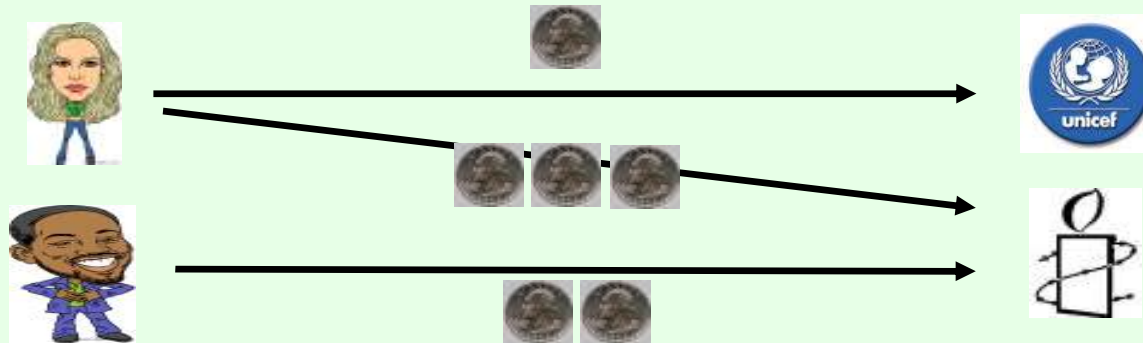
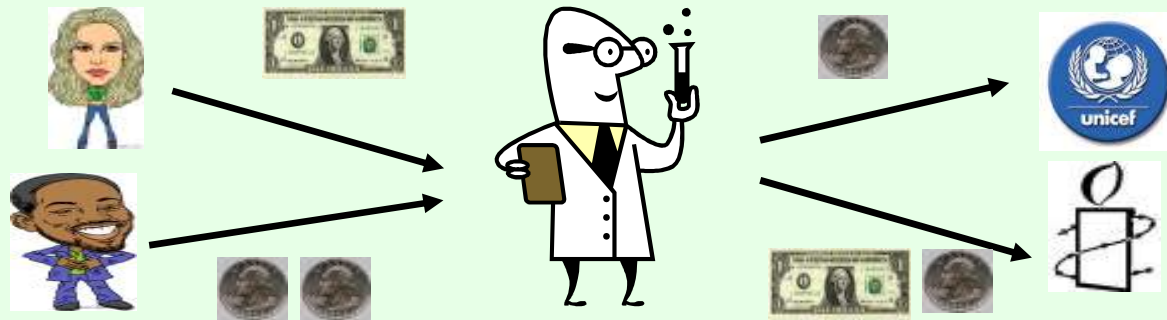
- $Total\ donated = x_1 + x_2 + \dots + x_m$



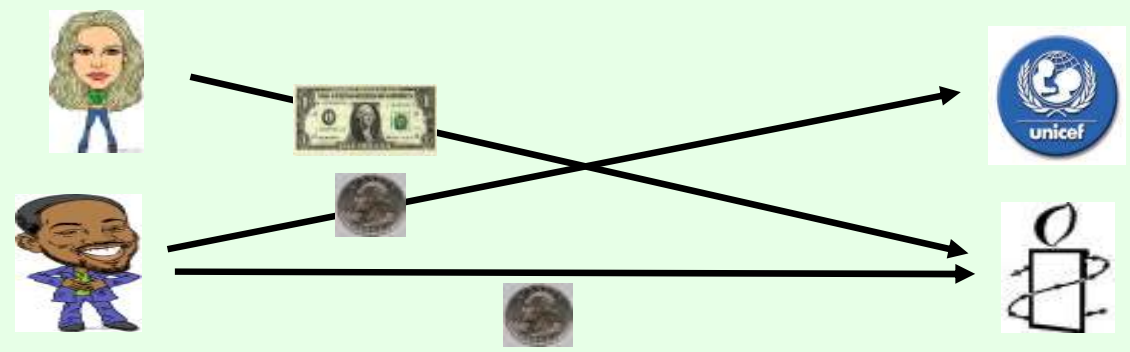
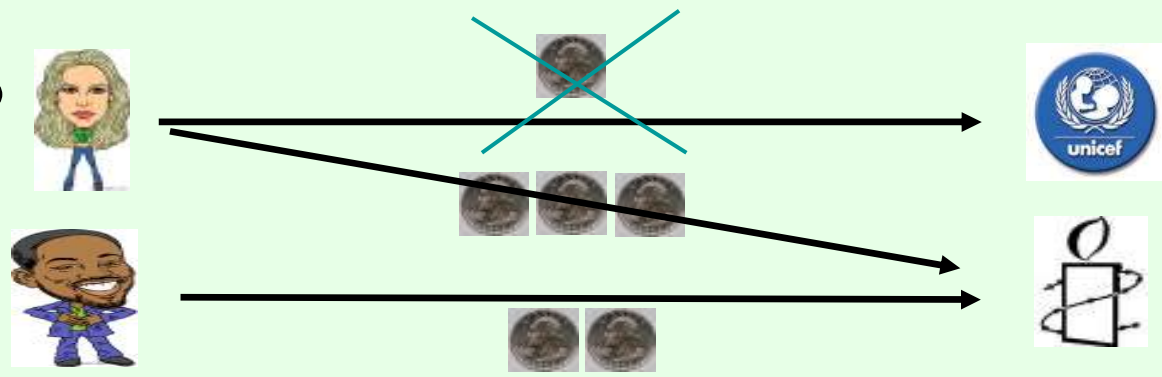
- $Surplus = y_1 + y_2 + \dots + y_n - x_1 - x_2 - \dots - x_m$



# Avoiding indirect payments



# No payments to disliked charities

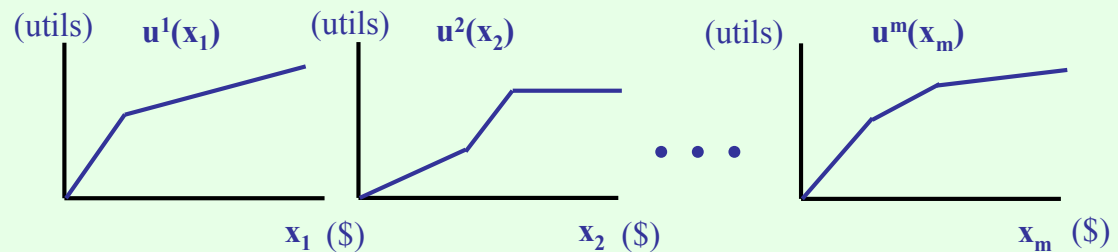
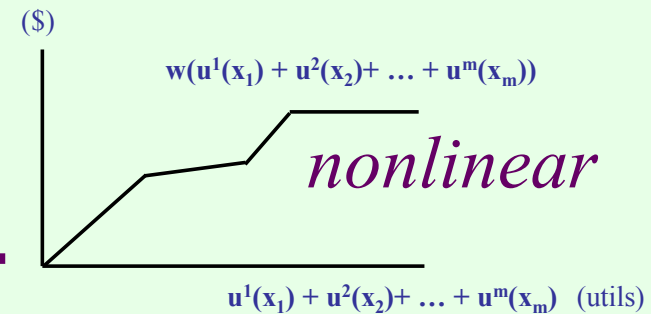


# Hardness of clearing

- NP-complete to decide if there exists a solution with objective  $> 0$
- That means: the problem is inapproximable to any ratio (unless  $P=NP$ )

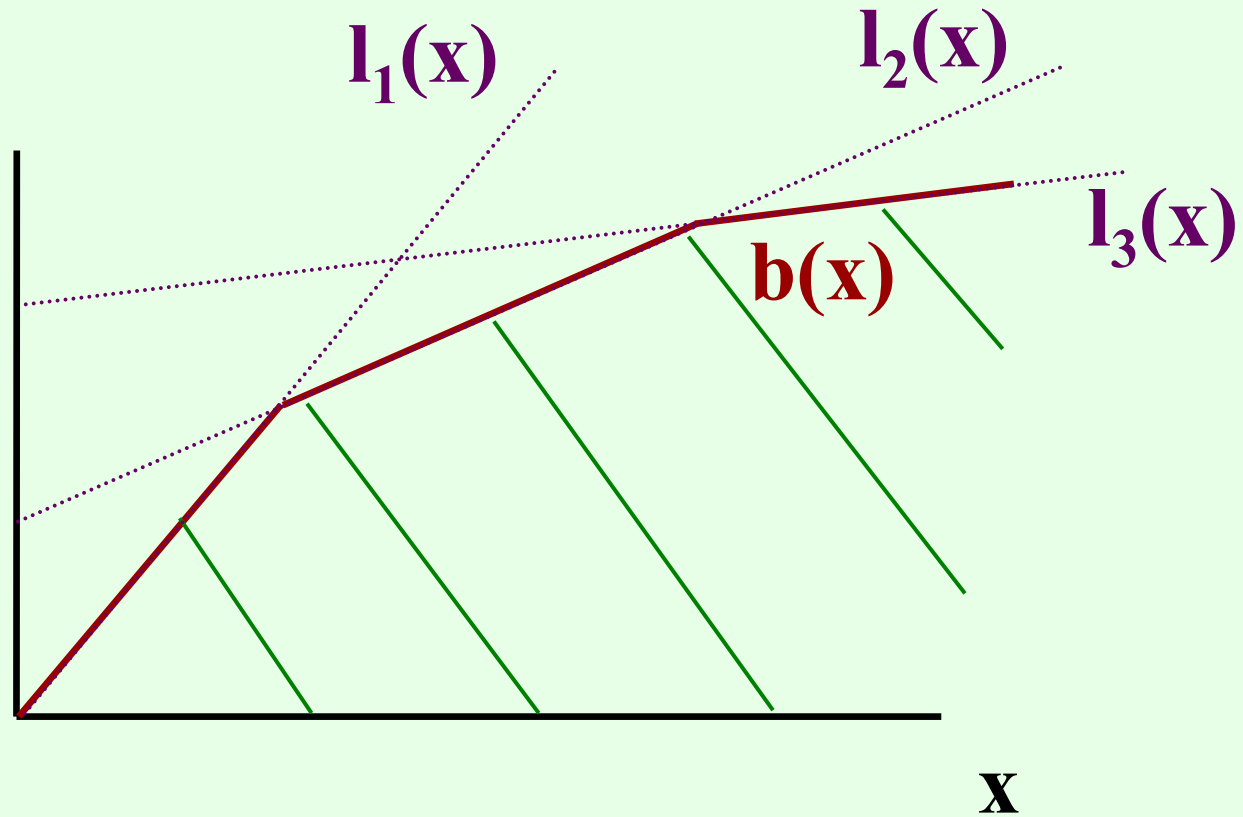
# General program formulation

- *Maximize*
  - $x_1 + x_2 + \dots + x_m$ , OR
  - $y_1 + y_2 + \dots + y_n - x_1 - x_2 - \dots - x_m$
- *Subject to*
  - $y_1 + y_2 + \dots + y_n - x_1 - x_2 - \dots - x_m \geq 0$
  - For all  $j$ :  $y_j \leq w_j(u_j^1 + u_j^2 + \dots + u_j^m)$
  - For all  $i, j$ :  $u_j^i \leq u_j^i(x_i)$



*nonlinear*

# Concave piecewise linear constraints



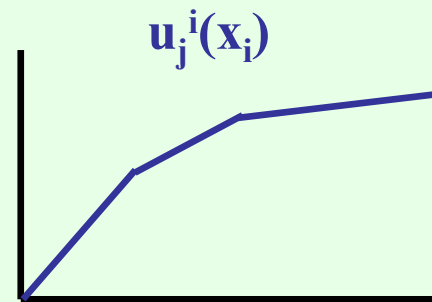
$$y \leq b(x) \quad \longrightarrow \quad \begin{aligned} y &\leq l_1(x) \\ y &\leq l_2(x) \\ y &\leq l_3(x) \end{aligned}$$



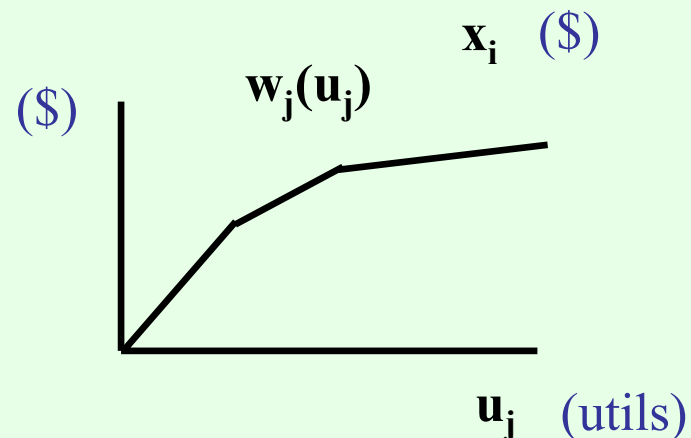
# Linear programming

- So, if all the bids are *concave*...

- All the  $u_j^i$  are concave (utils)



- All the  $w_j$  are concave



- Then the program is a linear program (solvable to optimality in polynomial time)
- Even if they are not concave, can solve as MIP