CPS 196.2
Learning in games

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Learning in (normal-form) games

- Approach we have taken so far when playing a game: just compute an optimal/equilibrium strategy
- Another approach: learn how to play a game by
  - playing it many times, and
  - updating your strategy based on experience
- Why?
  - Some of the game's utilities (especially the other players') may be unknown to you
  - The other players may not be playing an equilibrium strategy
  - Computing an optimal strategy can be hard
  - Learning is what humans typically do
  - …
- Learning strategies ~ strategies for the repeated game

Iterated best response

- In the first round, play something arbitrary
- In each following round, play a best response against what the other players played in the previous round
- If all players play this, it can converge (i.e. we reach an equilibrium) or cycle

0, 0	-1, 1	1, -1
1, -1	0, 0	-1, 1
-1, 1	1, -1	0, 0

rock-paper-scissors

-1, -1	0, 0
0, 0	-1, -1

a simple coordination game

- Alternating best response: players alternatingly change strategies: one player best-responds each odd round, the other best-responds each even round

Fictitious play

- In the first round, play something arbitrary
- In each following round, play a best response against the historical distribution of the other players' play
  - I.e. as if other players randomly select from their past actions
- Again, if this converges, we have a Nash equilibrium
- Can still fail to converge…

0, 0	-1, 1	1, -1
1, -1	0, 0	-1, 1
-1, 1	1, -1	0, 0

rock-paper-scissors

-1, -1	0, 0
0, 0	-1, -1

a simple congestion game

Does the historical distribution of play converge to equilibrium?

- … for iterated best response?
- … for fictitious play?

3, 0	1, 2
1, 2	2, 1

Historical distribution (non)convergence for fictitious play

- Historical distribution under fictitious play does not converge for Shapley's game (starting with (U, M)):

0, 0	0, 1	1, 0
1, 0	0, 0	0, 1
0, 1	1, 0	0, 0

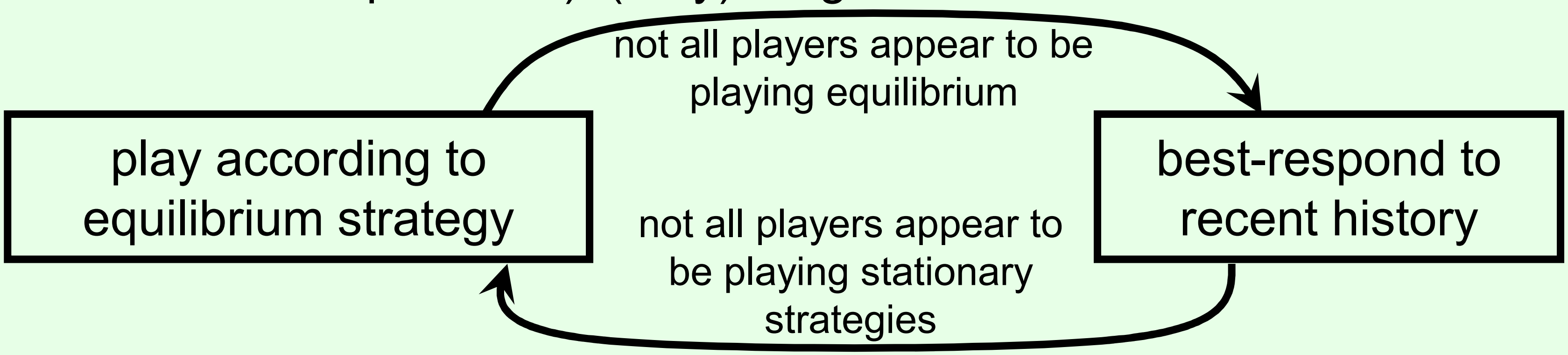
- The historical distribution under fictitious play converges for
  - generic 2x2 games [Miyasawa 61]
  - zero-sum games [Robinson 51]
  - games solvable by iterated strict dominance [Nachbar 90]

Regret

- For each player i, action a_i and time t, define the regret $r_i(a_i, t)$ as $(\Sigma_{1 \le t' \le t-1} u_i(a_i, a_{-i,t'}) - u_i(a_{i,t'}, a_{-i,t'}))/(t-1)$
- An algorithm has zero regret if for each a_i, the regret for a_i becomes nonpositive as t goes to infinity (almost surely) against any opponents
- Regret matching [Hart & Mas-Colell 00]: at time t, play an action that has positive regret $r_i(a_i, t)$ with probability proportional to $r_i(a_i, t)$
  - If none of the actions have positive regret, play uniformly at random
- Regret matching has zero regret
- If all players use regret matching, then play converges to the set of weak correlated equilibria
  - Weak correlated equilibrium: playing according to joint distribution is at least as good as any strategy that does not depend on the signal
- Variants of this converge to the set of correlated equilibria
- Smooth fictitious play [Fudenberg & Levine 95] also gives no regret
  - Instead of just best-responding to history, assign some small value to having a more “mixed” distribution

Targeted learning

- Assume that there is a limited set of possible opponents
- Try to do well against these
- Example: is there a learning algorithm that
  - learns to best-respond against any stationary opponent (one that always plays the same mixed strategy), and
  - converges to a Nash equilibrium (in actual strategies, not historical distribution) when playing against a copy of itself (so-called self-play)?
- [Bowling and Veloso AIJ02]: yes, if it is a 2-player 2x2 game and mixed strategies are observable
- [Conitzer and Sandholm ICML03/ML06]: yes (without those assumptions)
  - AWESOME algorithm (Adapt When Everybody is Stationary, Otherwise Move to Equilibrium): (very) rough sketch:



"Teaching"

- Suppose you are playing against a player that uses one of these strategies
  - Fictitious play, anything with no regret, AWESOME, …
- Also suppose you are very patient, i.e. you only care about what happens in the long run
- How will you (the row player) play in the following repeated games?
  - Hint: the other player will eventually best-respond to whatever you do

4, 4	3, 5
5, 3	0, 0

1, 0	3, 1
2, 1	4, 0

- Note relationship to optimal strategies to commit to
- There is some work on learning strategies that are in equilibrium with each other [Brafman & Tennenholtz AIJ04]

Evolutionary game theory

- Given: a symmetric game

	dove	hawk
dove	1, 1	0, 2
hawk	2, 0	-1, -1

Nash equilibria: (d, h), (h, d), ((.5, .5), (.5, .5))

- A large population of players plays this game, players are randomly matched to play with each other
- Each player plays a pure strategy
  - Fraction of players playing strategy s = p_s
  - p is vector of all fractions p_s (the state)
- Utility for playing s is $u(s, p) = \Sigma_{s'} p_{s'} u(s, s')$
- Players reproduce at a rate that is proportional to their utility, their offspring play the same strategy
  - Replicator dynamic
- $dp_s(t)/dt = p_s(t)(u(s, p(t)) - \Sigma_{s'} p_{s'} u(s', p(t)))$
- What are the steady states of this?

Stability

	dove	hawk
dove	1, 1	0, 2
hawk	2, 0	-1, -1

- A steady state is stable if slightly perturbing the state will not cause us to move far away from the state
- E.g. everyone playing dove is not stable, because if a few hawks are added their percentage will grow
- What about the mixed steady state?
- Proposition: every stable steady state is a Nash equilibrium of the symmetric game
- Slightly stronger criterion: a state is asymptotically stable if it is stable, and after slightly perturbing this state, we will (in the limit) return to this state

Evolutionarily stable strategies

- Now suppose players play mixed strategies
- A (single) mixed strategy σ is evolutionarily stable if the following is true:
  - Suppose all players play σ
  - Then, whenever a very small number of invaders enters that play a different strategy σ',
  - the players playing σ must get strictly higher utility than those playing σ' (i.e. σ must be able to repel invaders)
- σ will be evolutionarily stable if and only if for all σ'
  - $u(\sigma, \sigma) > u(\sigma', \sigma)$, or:
  - $u(\sigma, \sigma) = u(\sigma', \sigma)$ and $u(\sigma, \sigma') > u(\sigma', \sigma')$
- Proposition: every evolutionarily stable strategy is asymptotically stable under the replicator dynamic