CPS 196.2 - Intro to Computational Economics

Written Assignment 2: Repeated games, Bayesian games, GVA (due December 4 before class)

Please read the rules for assignments on the course web page. Contact Mingyu (mingyu@cs.duke.edu) or Vince (conitzer@cs.duke.edu) with any questions.

1. Repeated games.
Consider the following game:

4,5	8,3
6,2	2,6

Suppose that the game is infinitely repeated and that the limit-average payoff criterion is used. Using the folk theorem from class, state whether the following utility combinations can be maintained in a Nash equilibrium of the infinitely repeated game (that is, whether they are both feasible and enforceable). Justify your answers. (Hint: when determining what utilities are enforceable, keep in mind that the players can play mixed strategies. So, you should solve the zero-sum game that results from replacing player 2's utilities with the negative of player 1's utilities (for the case where 2 tries to punish 1, trying to minimize 1's utility), and vice versa. Also, not enforceable means that a player can guarantee herself a *strictly* higher utility.)

(a) $u_1 = 4, u_2 = 5$.
(b) $u_1 = 7, u_2 = 3.5$.
(c) $u_1 = 6, u_2 = 4$.
(d) $u_1 = 5, u_2 = 6$.

2. Funding a project.
There is a project that needs funding. Two agents can fund the project. Each agent has two choices: 1. Pay 1 to fund the project; 2. Pay 0. Depending on funding, the project may succeed or fail. If the total amount of money paid by the agents is 2, then the project will succeed with probability 1. If the total amount is 1, then the project will succeed with probability 1/3. If the total amount is 0, then the project will succeed with probability 0.

Each agent i ($i \in \{1, 2\}$) has a valuation v_i for a successful project. (Failed projects are worthless.) This valuation does not depend on whether the agent contributed or not (the agent can make use of the project regardless). Each valuation is drawn uniformly at random from $[0, 4]$.

For example, if agent 1 draws valuation 2 and pays 0, and agent 2 draws valuation 3 and pays 1, then:

- The probability of a successful project is 1/3;
- Agent 1's expected utility is $(1/3) \cdot 2 - 0 = 2/3$;
- Agent 2's expected utility is $(1/3) \cdot 3 - 1 = 0$.

(a) Show that the following strategy is a Bayes-Nash equilibrium of this game (if both players use it): pay 1 if and only if your valuation is at least 2. (Hint: show that if the player's valuation is 2, then she is indifferent between the two actions, given that the other player uses this strategy.)

(b) More generally, suppose that each player's valuation is drawn (independently) from some arbitrary distribution with cumulative density function F (but keep everything else the same). Show that the following is a Bayes-Nash equilibrium of this game (if both players use it): pay if and only if your valuation is at least x, where x is the solution to $F(x) \cdot x - 2x + 3 = 0$.

3. Generalized Vickrey Auction (= Clarke mechanism on combinatorial auctions).

Consider the following 3 bids in a combinatorial auction with 3 items (with free disposal):

$(\{a, b\}, 10)$ XOR $(\{c\}, 4)$

$(\{a, b\}, 6)$ XOR $(\{b, c\}, 9)$

$(\{a\}, 3)$ XOR $(\{a, b, c\}, 11)$

Solve the winner determination problem, and compute the GVA (Clarke) payments for the winning bidders. (Remember to remove each bidder's *entire* bid when calculating their payment.)

Instructions for turning in your homework: Turn in your writeup at the **beginning** of class. *Do not come to class late because you are still working on the homework!* If you know you will be late for class, turn in your homework earlier (e.g. under Vince's door, LSRC D207). You are welcome to go to Mingyu and Vince's office hours or to send them e-mail with questions.