

CPS 270: Artificial Intelligence

<http://www.cs.duke.edu/courses/fall08/cps270/>

Game Theory

Instructor: Vincent Conitzer

“2/3 of the average” game

- Everyone writes down a number between 0 and 100
- Person closest to $2/3$ of the average wins
- Example:
 - A says 50
 - B says 10
 - C says 90
 - Average(50, 10, 90) = 50
 - $2/3$ of average = 33.33
 - A is closest ($|50-33.33| = 16.67$), so A wins


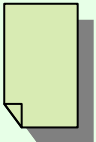

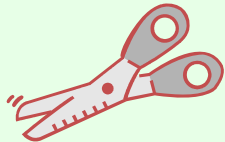
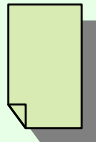

What is game theory?

- Game theory studies settings where multiple parties (**agents**) each have
 - different preferences (utility functions),
 - different actions that they can take
- Each agent's utility (potentially) depends on all agents' actions
 - What is optimal for one agent depends on what other agents do
 - Very circular!
- Game theory studies how agents can rationally form **beliefs** over what other agents will do, and (hence) how agents should **act**
 - Useful for acting as well as predicting behavior of others

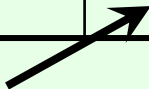
Rock-paper-scissors

Column player aka.
player 2
(simultaneously)
chooses a column

Row player
aka. player 1
chooses a row



0, 0	-1, 1	1, -1
1, -1	0, 0	-1, 1
-1, 1	1, -1	0, 0



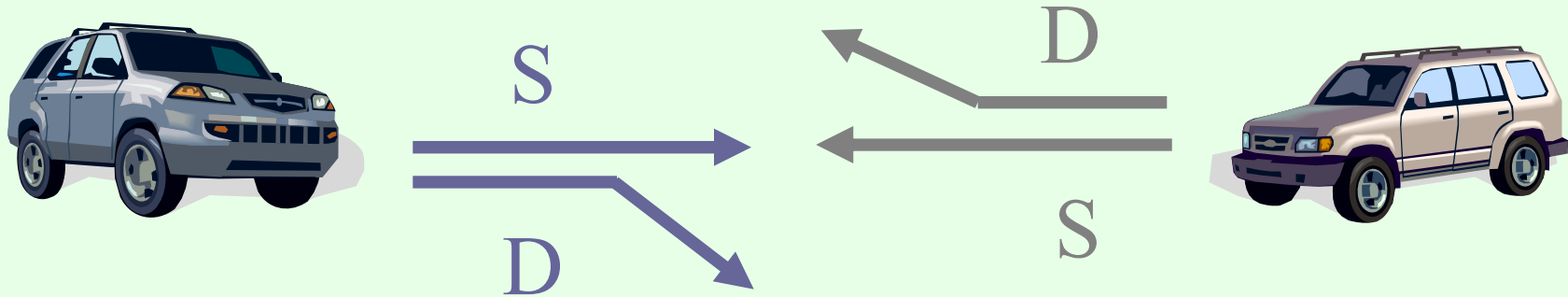
A row or column is
called an **action** or
(pure) strategy

Row player's utility is always listed first, column player's second

Zero-sum game: the utilities in each entry sum to 0 (or a constant)
Three-player game would be a 3D table with 3 utilities per entry, etc.

“Chicken”

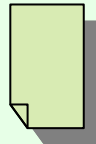
- Two players drive cars towards each other
- If one player goes straight, that player wins
- If both go straight, they both die



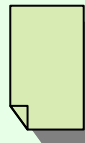
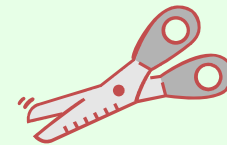
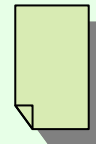
	D	S
D	0, 0	-1, 1
S	1, -1	-5, -5


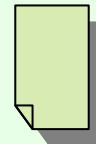
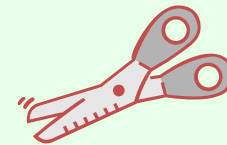

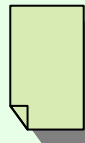

not zero-sum

Rock-paper-scissors – Seinfeld variant



MICKEY: All right, rock beats paper!
(Mickey smacks Kramer's hand for losing)
KRAMER: I thought paper covered rock.
MICKEY: Nah, rock flies right through paper.
KRAMER: What beats rock?
MICKEY: (looks at hand) Nothing beats rock.


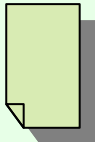


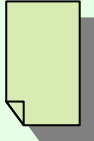



			
	0, 0	1, -1	1, -1
	-1, 1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

Dominance

- Player i 's strategy s_i **strictly dominates** s_i' if
 - for any s_{-i} , $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$
- s_i **weakly dominates** s_i' if
 - for any s_{-i} , $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$; and
 - for some s_{-i} , $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$

-i = "the player(s) other than i"

			
	0, 0	1, -1	1, -1
	-1, 1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

Prisoner's Dilemma

- Pair of criminals has been caught
- District attorney has evidence to convict them of a minor crime (1 year in jail); knows that they committed a major crime together (3 years in jail) but cannot prove it
- Offers them a deal:
 - If both confess to the major crime, they each get a 1 year reduction
 - If only one confesses, that one gets 3 years reduction

	confess	don't confess
confess	-2, -2	0, -3
don't confess	-3, 0	-1, -1

“Should I buy an SUV?”

purchasing cost

accident cost



cost: 5

cost: 5



cost: 5



cost: 3

cost: 8



cost: 2

cost: 5


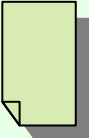



cost: 5



-10, -10	-7, -11
-11, -7	-8, -8

Mixed strategies

- **Mixed strategy** for player i = **probability distribution** over player i 's (pure) strategies
- E.g. $1/3$  , $1/3$  , $1/3$ 
- Example of dominance by a mixed strategy:

$1/2$	3, 0	0, 0
$1/2$	0, 0	3, 0
	1, 0	1, 0

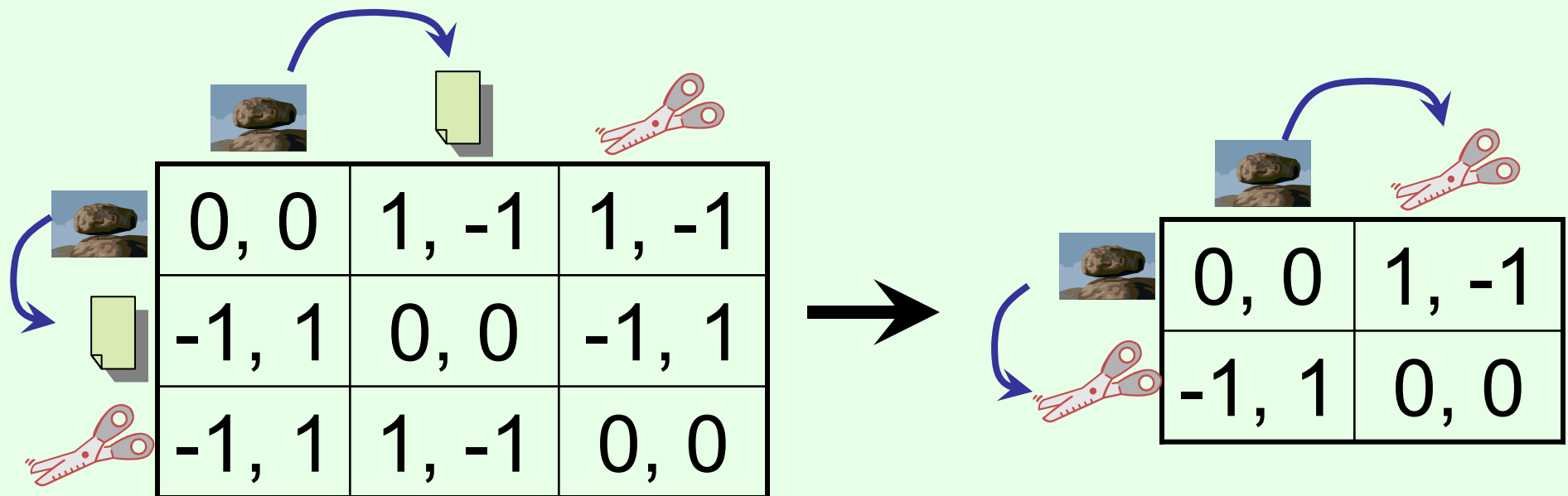
A blue bracket on the left side of the table groups the first two rows, with a curved arrow pointing from the bracket to the third row, indicating that the mixed strategy of the first two rows dominates the third row.

Checking for dominance by mixed strategies

- Linear program for checking whether strategy s_i^* is **strictly** dominated by a mixed strategy:
 - maximize ε
 - such that:
 - for any s_{-i} , $\sum_{s_i} \mathbf{p}_{s_i} u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i}) + \varepsilon$
 - $\sum_{s_i} \mathbf{p}_{s_i} = 1$
- Linear program for checking whether strategy s_i^* is **weakly** dominated by a mixed strategy:
 - maximize $\sum_{s_{-i}} (\sum_{s_i} \mathbf{p}_{s_i} u_i(s_i, s_{-i})) - u_i(s_i^*, s_{-i})$
 - such that:
 - for any s_{-i} , $\sum_{s_i} \mathbf{p}_{s_i} u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i})$
 - $\sum_{s_i} \mathbf{p}_{s_i} = 1$

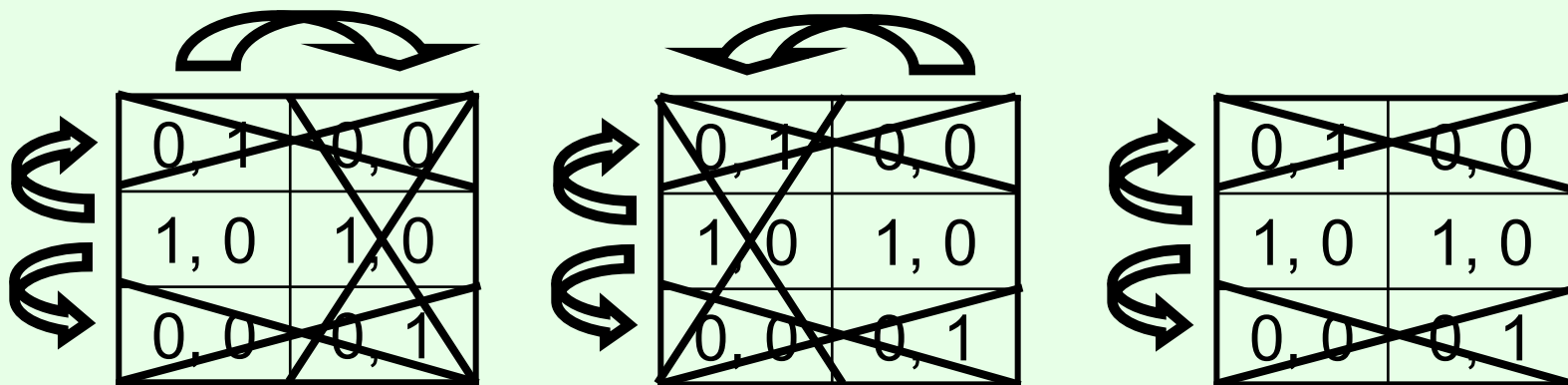
Iterated dominance

- Iterated dominance: remove (strictly/weakly) dominated strategy, repeat
- Iterated strict dominance on Seinfeld's RPS:



Iterated dominance: path (in)dependence

Iterated weak dominance is **path-dependent**:
sequence of eliminations may determine which
solution we get (if any)
(whether or not dominance by mixed strategies allowed)



Iterated strict dominance is **path-independent**: elimination
process will always terminate at the same point
(whether or not dominance by mixed strategies allowed)

“2/3 of the average” game revisited

