

## Homework 2: First-order logic (due October 7 before class)

Please read the rules for assignments on the course web page. Contact Lirong (lxia@cs.duke.edu) or Vince (conitzer@cs.duke.edu) with any questions.

### 1 (40 points).

Convert the following English sentences to first-order logic.

- Every continent that is not Australia or Antarctica is connected to another continent.
- Every person who is smart and studies hard will get a higher score than every person who is not smart and does not study hard.
- Every thing that walks like a duck and talks like a duck is either a duck or a human imitating a duck.
- A gold medal is worth more than a silver medal, if they are medals in the same event.
- Every thing that loves all humans is a dog.
- There is a dog that does not love all humans.
- Every thing that is an enemy of some thing that is an enemy of me is a friend of me (“the enemy of my enemy is my friend”).
- There are at least two points on the world such that if from that point  $x$ , you travel one meter north, then one meter east, and then one meter south, you are back at point  $x$ . (Unrelated bonus question (0 points): what is the set of all of these points?)

### 2 (20 points).

Apply resolution to obtain the most general conclusion possible.

- $\forall x, y : \text{LovesTheCombinationOf}(\text{John}, x, y) \vee \text{MakesSick}(x, \text{John}) \vee \text{RuinsTasteOf}(y, x)$
- $\forall v, w : \neg \text{LovesTheCombinationOf}(v, \text{Rice}, w) \vee \text{Flavorful}(w)$

### 3 (40 points).

Consider again the phrase “the enemy of my enemy is my friend” (stated more precisely above). Suppose this is true not just for me, but for every thing, that is, for any  $x$ , any enemy of any enemy of  $x$  is a friend of  $x$ . Also suppose that if  $x$  is an enemy of  $y$ , then  $y$  is an enemy of  $x$ . Furthermore, suppose that every  $x$  has at least two enemies. Convert these sentences into first-order logic, and formally prove that every  $x$  has at least one friend (other than itself). You may use any reasoning pattern, but try to make your reasoning patterns as basic and formal as possible.