Homework 5:
Numerical Integration of PDEs and Its Applications

Due date : Dec. 10, 2010

1. Observe a numerical issue in the computation following the particular linear difference equation, which may be resulted from discretizing an ODE,

\[ x_{j+1} = x_j + h(w_0 x_j + w_1 x_{j-1}) = 2.25x_j - 0.5x_{j-1}, \quad j > 2 \]
\[ x(0) = x_0, \quad x(1) = x_1, \quad \text{I.V.} \]

(a) In analysis, verify that the two sequences with initial values \([x_0, x_1] = [1/3, 1/12]\) and \([x_0, x_1] = [1, 1/4]\), respectively, are linearly proportional.
(b) Observe and describe the difference in the numerical sequences up to \(j = 60\), for example.
(c) Find the cause(s) and explanation(s) for the difference.
(d) Propose an approach for a stable or stabilized computation.

2. The FHN switch model. cf. FHNswitchXX.m

(a) Adjust the three states from \([-1, 0, 1]\) to \([0, \tau, 1]\), where \(\tau\) is a threshold not necessarily equal to 1/2;
(b) Try to identify certain behaviors of the switch as its characteristics;
(c) Find a time step that is relatively large and preserves the switch characteristics with each of the three methods.
(d) Describe briefly the integration rule underlying the Forward Euler method;
(e) Describe briefly the integration rule underlying the Backward Euler method, and the difference at each integration step from the Forward Euler method;
(f) Describe briefly the temporal integration procedure based on the trapezoidal rule, and its difference from the backward Euler method.
3. The FHN oscillator model. cf. FHNoScillator.m

(a) Try to identify certain behaviors of the oscillator as its characteristics. Find a time step that is relatively large and preserves the oscillator characteristics.

4. The FHN cell arrays. cf. FHNcellArray1d.m, FHNcellRing.m, FHNcellArray2d.m

(a) Observe and describe the difference in 1D propagation between the refractory boundary condition and the circular/periodic boundary condition, with the same model parameters and initial conditions.

(b) Provide FHNcellTorus2d.m and produce a snapshot for 2D propagation with the torus or periodic boundary condition, using the same model parameters and initial conditions as with the refractory condition.