CPS 296.1
Utility theory

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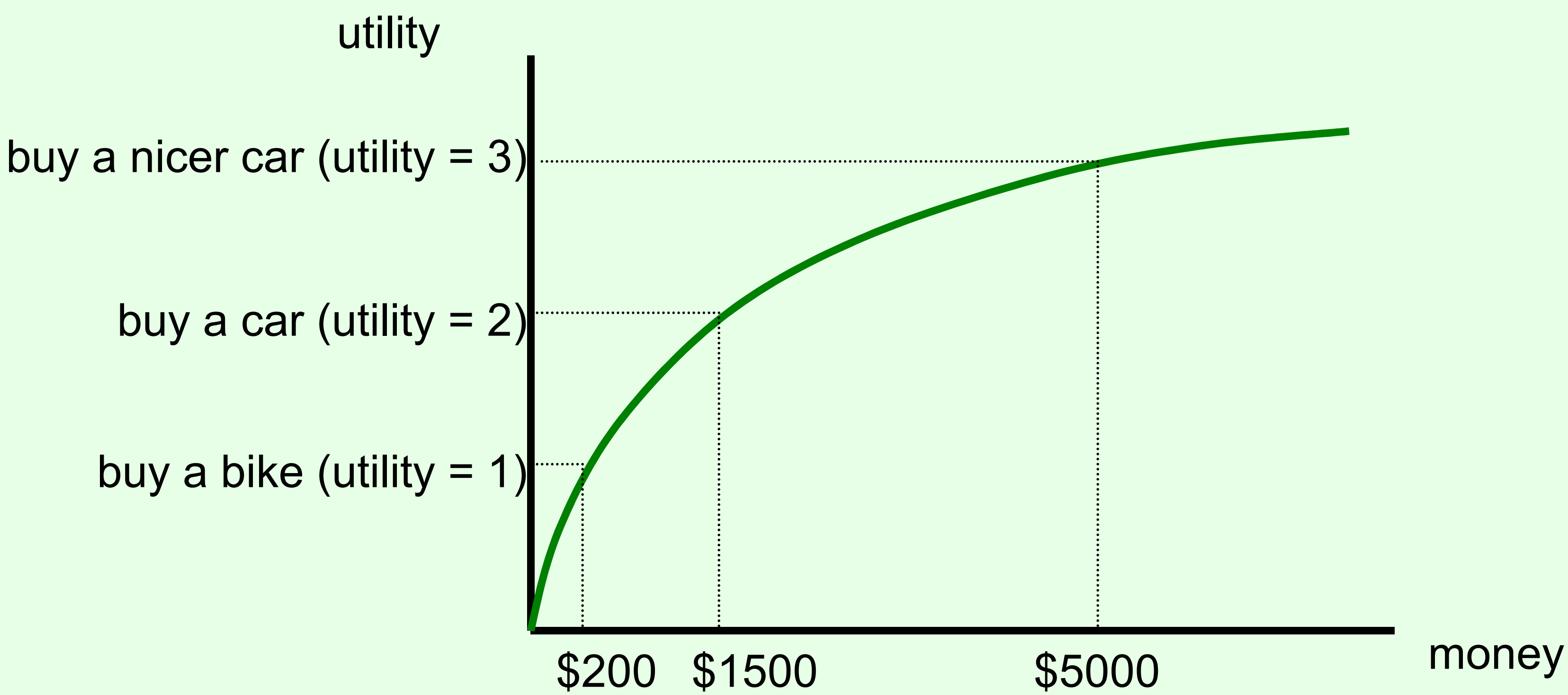
Risk attitudes

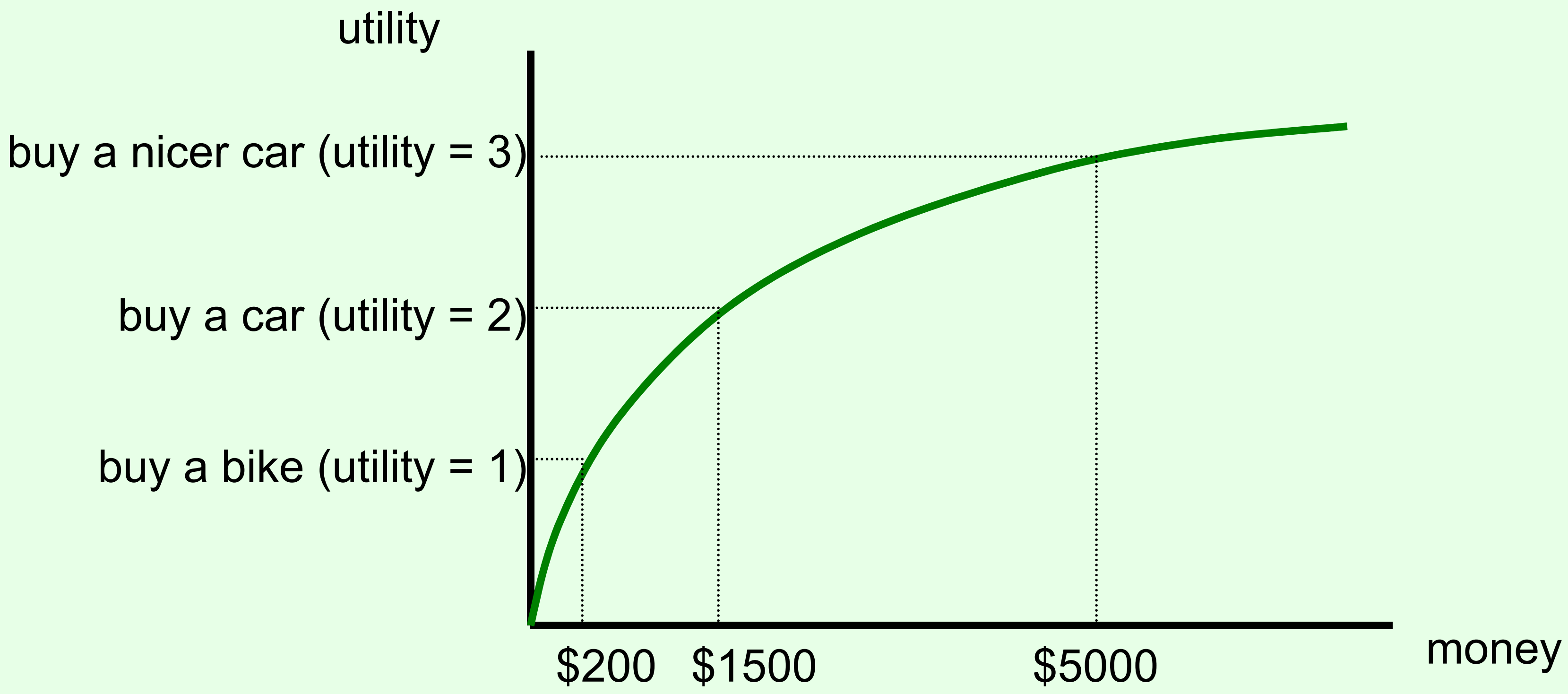
- Which would you prefer?
  - A lottery ticket that pays out \$10 with probability .5 and \$0 otherwise, or
  - A lottery ticket that pays out \$3 with probability 1
- How about:
  - A lottery ticket that pays out \$100,000,000 with probability .5 and \$0 otherwise, or
  - A lottery ticket that pays out \$30,000,000 with probability 1
- Usually, people do not simply go by expected value
- An agent is risk-neutral if she only cares about the expected value of the lottery ticket
- An agent is risk-averse if she always prefers the expected value of the lottery ticket to the lottery ticket
  - Most people are like this
- An agent is risk-seeking if she always prefers the lottery ticket to the expected value of the lottery ticket

Decreasing marginal utility

- Typically, at some point, having an extra dollar does not make people much happier (decreasing marginal utility)

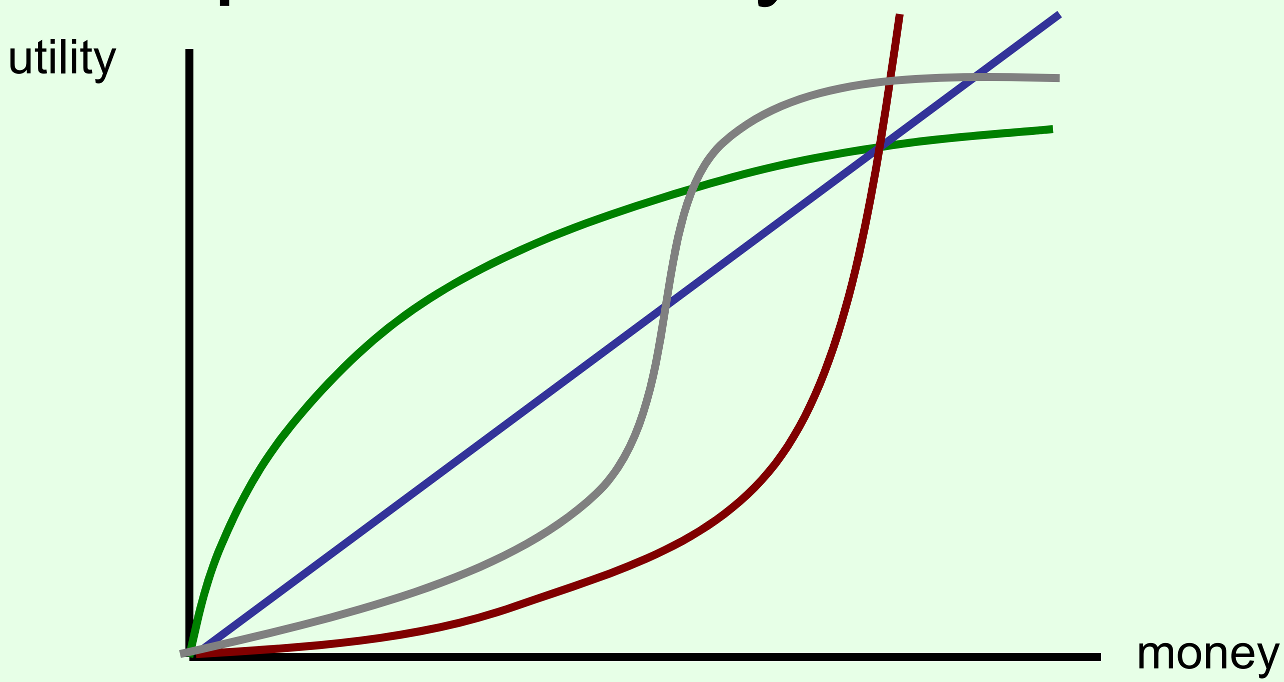


Maximizing expected utility



- Lottery 1: get $1500 with probability 1
  - gives expected utility 2
- Lottery 2: get $5000 with probability .4, $200 otherwise
  - gives expected utility .4*3 + .6*1 = 1.8
  - (expected amount of money = .4*$5000 + .6*$200 = $2120 > $1500)
- So: maximizing expected utility is consistent with risk aversion

Different possible risk attitudes under expected utility maximization



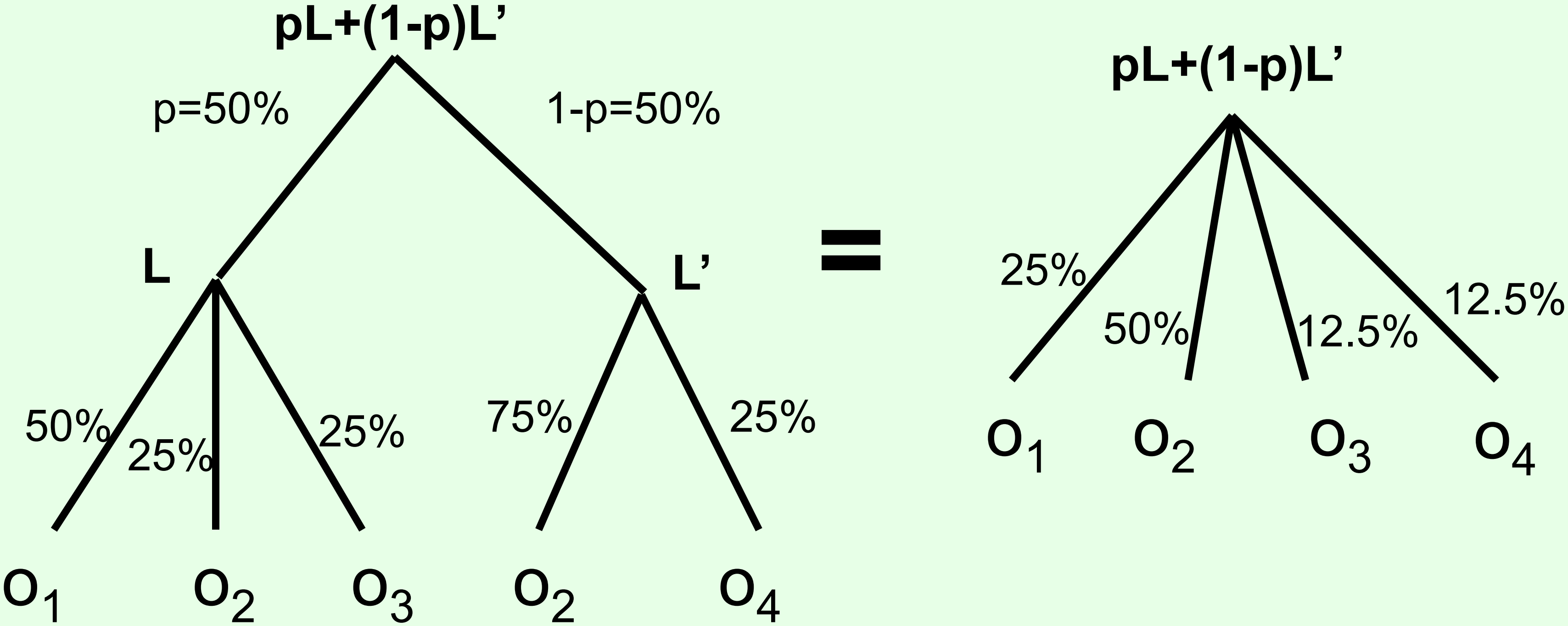
- Green has decreasing marginal utility → risk-averse
- Blue has constant marginal utility → risk-neutral
- Red has increasing marginal utility → risk-seeking
- Grey's marginal utility is sometimes increasing, sometimes decreasing → neither risk-averse (everywhere) nor risk-seeking (everywhere)

What is utility, anyway?

- Function u: O $\rightarrow \Re$ (O is the set of "outcomes" that lotteries randomize over)
- What are its units?
  - It doesn't really matter
  - If you replace your utility function by u'(o) = a + bu(o), your behavior will be unchanged
- Why would you want to maximize expected utility?
  - This is a question about preferences over lotteries

Compound lotteries

- For two lottery tickets L and L’, let pL + (1-p)L’ be the “compound” lottery ticket where you get lottery ticket L with probability p, and L’ with probability 1-p



Sufficient conditions for expected utility

- L ≥ L' means that L is (weakly) preferred to L'
  - (≥ should be complete, transitive)
- **Expected utility theorem.** Suppose
  - (continuity axiom) for all L, L', L'', {p: pL + (1-p)L' ≥ L''} and {p: pL + (1-p)L' ≤ L''} are closed sets,
  - (independence axiom – more controversial) for all L, L', L'', p, we have L ≥ L' if and only if pL + (1-p)L'' ≥ pL' + (1-p)L''

  then there exists a function $u: O \rightarrow \Re$ so that L ≥ L' if and only if L gives a higher expected value of u than L'