SQL: Recursion

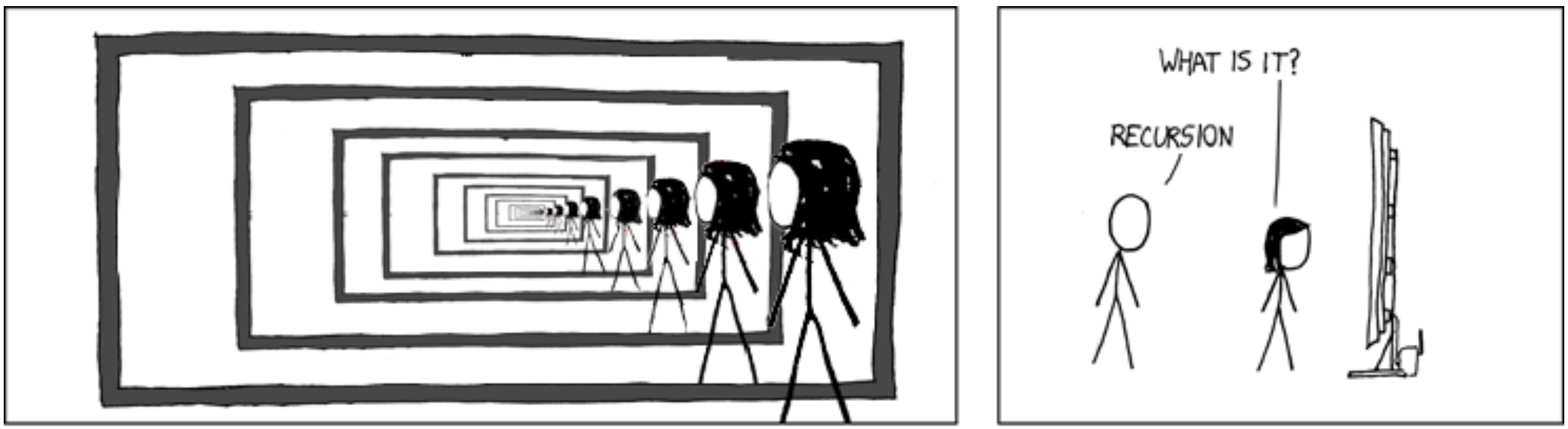
Introduction to Databases

CompSci 316 Fall 2017

DUKE
COMPUTER SCIENCE

Announcements (Tue., Oct. 3)

- Homework #2 due tonight
  - Deadline extended to Thursday for Problem 6 (Gradiance) only
- Midterm in class Thursday
  - Open-book, open-notes
  - Same format as sample midterm
    - Sample solution also posted in Sakai
- Project Milestone #1 due next Thursday

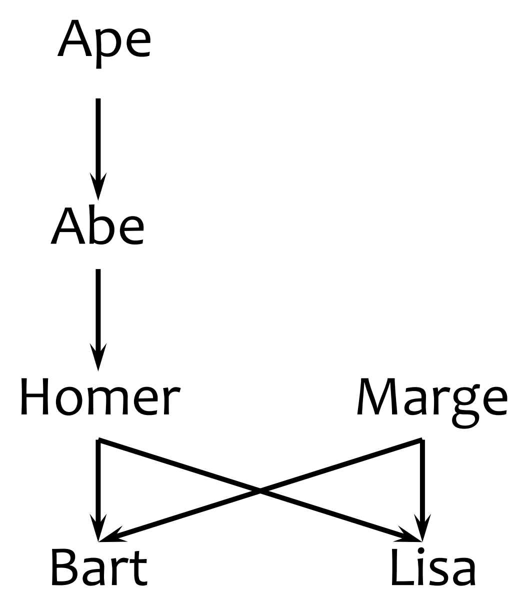


http://xkcdsw.com/1105

A motivating example

Parent (parent, child)

parent	*child*
`Homer`	`Bart`
`Homer`	`Lisa`
`Marge`	`Bart`
`Marge`	`Lisa`
`Abe`	`Homer`
`Ape`	`Abe`



- Example: find Bart's ancestors
- "Ancestor" has a recursive definition
  - X is Y's ancestor if
    - X is Y's parent, or
    - X is Z's ancestor and Z is Y's ancestor

Recursion in SQL

- SQL2 had no recursion
  - You can find Bart's parents, grandparents, great grandparents, etc.

```
SELECT p1.parent AS grandparent
FROM Parent p1, Parent p2
WHERE p1.child = p2.parent
AND p2.child = 'Bart';
```

  - But you cannot find all his ancestors with a single query
- SQL3 introduces recursion
  - `WITH` clause
  - Implemented in PostgreSQL (common table expressions)

Ancestor query in SQL3

```
WITH RECURSIVE
Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
SELECT anc
FROM Ancestor
WHERE desc = 'Bart';
```

base case

recursion step

Define a relation recursively

Query using the relation defined in `WITH` clause

Fixed point of a function

- If $f : T \rightarrow T$ is a function from a type T to itself, a fixed point of f is a value x such that $f(x) = x$
- Example: What is the fixed point of $f(x) = x/2$?
  - 0, because $f(0) = 0/2 = 0$
- To compute a fixed point of f
  - Start with a "seed": $x \leftarrow x_0$
  - Compute $f(x)$
    - If $f(x) = x$, stop; x is fixed point of f
    - Otherwise, $x \leftarrow f(x)$; repeat
- Example: compute the fixed point of $f(x) = x/2$
  - With seed 1: 1, 1/2, 1/4, 1/8, 1/16, … → 0

☞ Doesn't always work, but happens to work for us!

Fixed point of a query

- A query q is just a function that maps an input table to an output table, so a fixed point of q is a table T such that $q(T) = T$
- To compute fixed point of q
  - Start with an empty table: $T \leftarrow \emptyset$
  - Evaluate q over T
    - If the result is identical to T, stop; T is a fixed point
    - Otherwise, let T be the new result; repeat

  ☞Starting from ∅ produces the unique minimal fixed point (assuming q is monotone)

Finding ancestors

- ```
  WITH RECURSIVE
  Ancestor(anc, desc) AS
  ((SELECT parent, child FROM Parent)
   UNION
   (SELECT a1.anc, a2.desc
    FROM Ancestor a1, Ancestor a2
    WHERE a1.desc = a2.anc))
  ```
  - Think of the definition as *Ancestor* = *q*(*Ancestor*)

parent	*child*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	*desc*

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa
  ```

Intuition behind fixed-point iteration

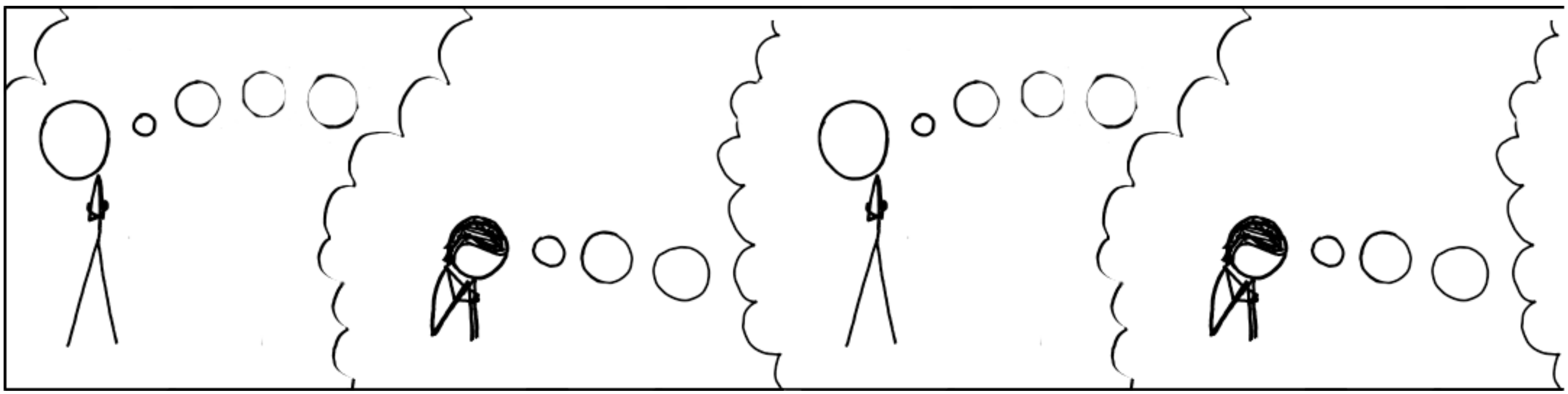
- Initially, we know nothing about ancestor-descendent relationships
- In the first step, we deduce that parents and children form ancestor-descendent relationships
- In each subsequent steps, we use the facts deduced in previous steps to get more ancestor-descendent relationships
- We stop when no new facts can be proven

Linear recursion

- With linear recursion, a recursive definition can make only one reference to itself
- Non-linear
  - ```
    WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent)
     UNION
     (SELECT a1.anc, a2.desc
      FROM Ancestor a1, Ancestor a2
      WHERE a1.desc = a2.anc))
    ```
- Linear
  - ```
    WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent)
     UNION
     (SELECT anc, child
      FROM Ancestor, Parent
      WHERE desc = parent))
    ```

Linear vs. non-linear recursion

- Linear recursion is easier to implement
  - For linear recursion, just keep joining newly generated *Ancestor* rows with *Parent*
  - For non-linear recursion, need to join newly generated *Ancestor* rows with all existing *Ancestor* rows
- Non-linear recursion may take fewer steps to converge, but perform more work
  - Example: $a \to b \to c \to d \to e$
  - Linear recursion takes 4 steps
  - Non-linear recursion takes 3 steps
    - More work: e.g., $a \to d$ has two different derivations



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Mutual recursion example

- Table *Natural* (*n*) contains 1, 2, …, 100
- Which numbers are even/odd?
  - An odd number plus 1 is an even number
  - An even number plus 1 is an odd number
  - 1 is an odd number

```
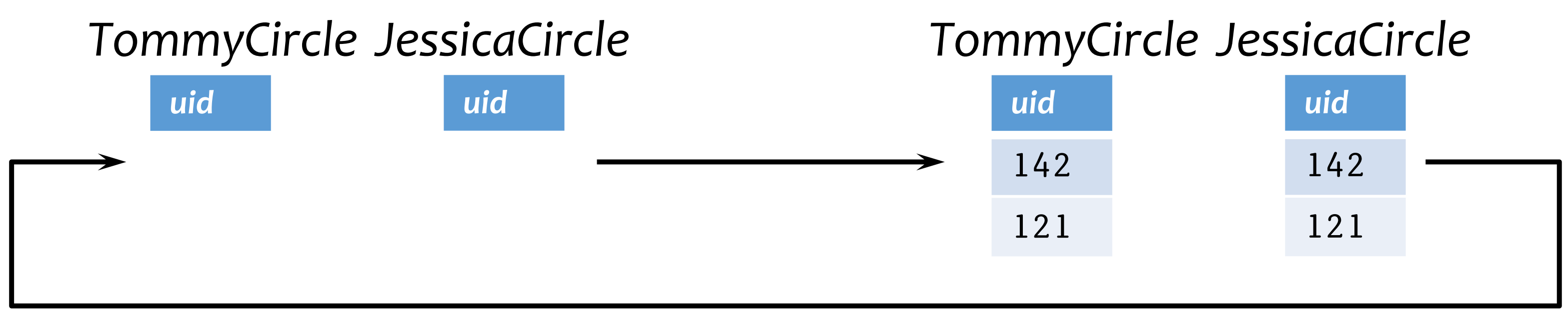
WITH RECURSIVE Even(n) AS
     (SELECT n FROM Natural
      WHERE n = ANY(SELECT n+1 FROM Odd)),
     RECURSIVE Odd(n) AS
     ((SELECT n FROM Natural WHERE n = 1)
      UNION
      (SELECT n FROM Natural
       WHERE n = ANY(SELECT n+1 FROM Even)))
```

Semantics of `WITH`

- `WITH RECURSIVE` R_1 `AS` Q_1`, …,`
  `RECURSIVE` R_n `AS` Q_n
  Q`;`
  - Q and $Q_1, \dots, Q_n$ may refer to $R_1, \dots, R_n$
- Semantics
  1. $R_1 \leftarrow \emptyset, \dots, R_n \leftarrow \emptyset$
  2. Evaluate $Q_1, \dots, Q_n$ using the current contents of $R_1, \dots, R_n$:
     $R_1^{new} \leftarrow Q_1, \dots, R_n^{new} \leftarrow Q_n$
  3. If $R_i^{new} \neq R_i$ for some i
     3.1. $R_1 \leftarrow R_1^{new}, \dots, R_n \leftarrow R_n^{new}$
     3.2. Go to 2.
  4. Compute Q using the current contents of $R_1, \dots R_n$
     and output the result

Computing mutual recursion

```
WITH RECURSIVE Even(n) AS
       (SELECT n FROM Natural
        WHERE n = ANY(SELECT n+1 FROM Odd)),
      RECURSIVE Odd(n) AS
       ((SELECT n FROM Natural WHERE n = 1)
        UNION
        (SELECT n FROM Natural
         WHERE n = ANY(SELECT n+1 FROM Even)))
```

- *Even* = ∅, *Odd* = ∅
- *Even* = ∅, *Odd* = {1}
- *Even* = {2}, *Odd* = {1}
- *Even* = {2}, *Odd* = {1, 3}
- *Even* = {2, 4}, *Odd* = {1, 3}
- *Even* = {2, 4}, *Odd* = {1, 3, 5}
- …

Fixed points are not unique

```
WITH RECURSIVE
Ancestor(anc, desc) AS
((SELECT parent, child FROM
Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
```

parent	*child*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	*desc*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa
Bogus	Bogus

Note how the bogus tuple reinforces itself!

- But if q is monotone, then all these fixed points must contain the fixed point we computed from fixed-point iteration starting with ∅
  - Thus the unique minimal fixed point is the “natural” answer

Mixing negation with recursion

- If q is non-monotone
  - The fixed-point iteration may flip-flop and never converge
  - There could be multiple minimal fixed points—we wouldn't know which one to pick as answer!
- Example: popular users (pop $\geq$ 0.8) join either Jessica's Circle or Tommy's
  - Those not in Jessica's Circle should be in Tom's
  - Those not in Tom's Circle should be in Jessica's
  - ```
    WITH RECURSIVE TommyCircle(uid) AS
         (SELECT uid FROM User WHERE pop >= 0.8
          AND uid NOT IN (SELECT uid FROM JessicaCircle)),
         RECURSIVE JessicaCircle(uid) AS
         (SELECT uid FROM User WHERE pop >= 0.8
          AND uid NOT IN (SELECT uid FROM TommyCircle))
    ```
    ```

Fixed-point iter may not converge

```
WITH RECURSIVE TommyCircle(uid) AS
     (SELECT uid FROM User WHERE pop >= 0.8
      AND uid NOT IN (SELECT uid FROM JessicaCircle)),
     RECURSIVE JessicaCircle(uid) AS
     (SELECT uid FROM User WHERE pop >= 0.8
      AND uid NOT IN (SELECT uid FROM TommyCircle))
```

uid	*name*	*age*	*pop*
142	Bart	10	0.9
121	Allison	8	0.85



Multiple minimal fixed points

```
WITH RECURSIVE TommyCircle(uid) AS
     (SELECT uid FROM User WHERE pop >= 0.8
      AND uid NOT IN (SELECT uid FROM JessicaCircle)),
     RECURSIVE JessicaCircle(uid) AS
     (SELECT uid FROM User WHERE pop >= 0.8
      AND uid NOT IN (SELECT uid FROM TommyCircle))
```

uid	name	age	pop
142	Bart	10	0.9
121	Allison	8	0.85

TommyCircle

uid
142

JessicaCircle

uid
121

TommyCircle

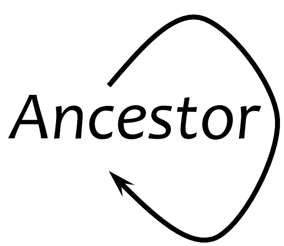
uid
121

JessicaCircle

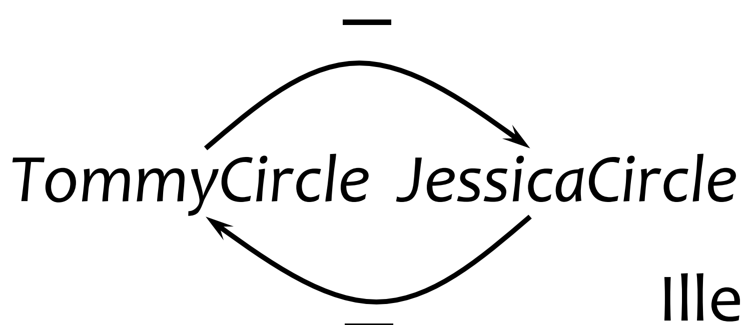
uid
142

Legal mix of negation and recursion

- Construct a dependency graph
  - One node for each table defined in `WITH`
  - A directed edge $R \rightarrow S$ if R is defined in terms of S
  - Label the directed edge "−" if the query defining R is not monotone with respect to S
- Legal SQL3 recursion: no cycle with a "−" edge
  - Called stratified negation
- Bad mix: a cycle with at least one edge labeled "−"



Legal!

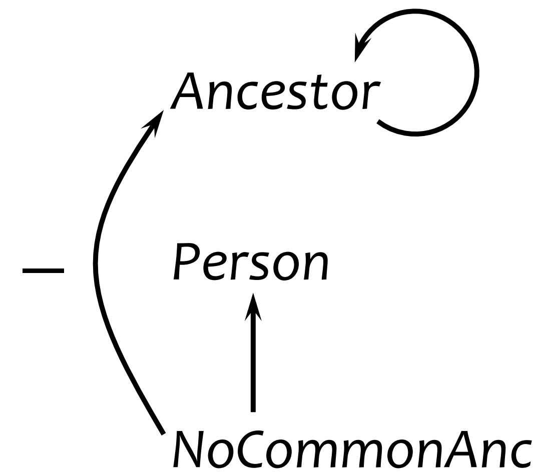


Illegal!

Stratified negation example

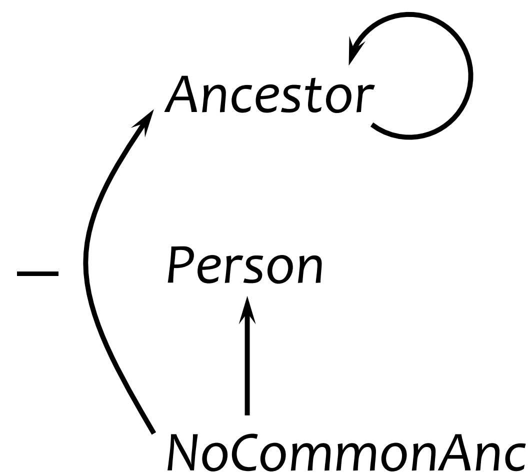
- Find pairs of persons with no common ancestors

```
WITH RECURSIVE Ancestor(anc, desc) AS
     ((SELECT parent, child FROM Parent) UNION
      (SELECT a1.anc, a2.desc
       FROM Ancestor a1, Ancestor a2
       WHERE a1.desc = a2.anc)),

     Person(person) AS
     ((SELECT parent FROM Parent) UNION
      (SELECT child FROM Parent)),

     NoCommonAnc(person1, person2) AS
     ((SELECT p1.person, p2.person
       FROM Person p1, Person p2
       WHERE p1.person <> p2.person)
      EXCEPT
      (SELECT a1.desc, a2.desc
       FROM Ancestor a1, Ancestor a2
       WHERE a1.anc = a2.anc))

SELECT * FROM NoCommonAnc;
```



Evaluating stratified negation

- The stratum of a node R is the maximum number of "—" edges on any path from R in the dependency graph
  - *Ancestor*: stratum 0
  - *Person*: stratum 0
  - *NoCommonAnc*: stratum 1



- Evaluation strategy
  - Compute tables lowest-stratum first
  - For each stratum, use fixed-point iteration on all nodes in that stratum
    - Stratum 0: *Ancestor* and *Person*
    - Stratum 1: *NoCommonAnc*

☞ Intuitively, there is no negation within each stratum

Summary

- SQL3 `WITH` recursive queries
- Solution to a recursive query (with no negation): unique minimal fixed point
- Computing unique minimal fixed point: fixed-point iteration starting from ∅
- Mixing negation and recursion is tricky
    - Illegal mix: fixed-point iteration may not converge; there may be multiple minimal fixed points
    - Legal mix: stratified negation (compute by fixed-point iteration stratum by stratum)