Recursive Query Evaluation and Datalog

Instructor: Sudeepa Roy
Announcements

• HW3 due Monday 11/26

• Next week
  – practice pop-up quiz on transactions (all lectures)

• Project presentation in last class, but final report due 2 days before final
Where are we now?

We learnt
- Relational Model and Query Languages
  - SQL, RA, RC
  - Postgres (DBMS)
    - HW1
- Database Normalization
- DBMS Internals
  - Storage
  - Indexing
  - Query Evaluation
  - Operator Algorithms
  - External sort
  - Query Optimization
- Map-reduce and spark
  - HW2

- Transactions
  - Basic concepts
  - Concurrency control
  - Recovery
- Distributed DBMS
- NOSQL
- Parallel DBMS
Today

• Semantic of recursion in databases

• Datalog
  – for recursion in database queries

• Views
Recursion!

http://xkcdsw.com/1105
A motivating example

**Example**: find Bart’s ancestors

“**Ancestor**” has a recursive definition

– $X$ is $Y$’s ancestor if
  – $X$ is $Y$’s parent, or
  – $X$ is $Z$’s ancestor and $Z$ is $Y$’s ancestor
Recursion in SQL

• SQL2 had no recursion

  – You can find Bart’s parents, grandparents, great grandparents, etc.

    \[
    \text{SELECT p1.parent AS grandparent} \\
    \text{FROM Parent p1, Parent p2} \\
    \text{WHERE p1.child = p2.parent} \\
    \text{AND p2.child = 'Bart'};
    \]

  – But you cannot find all his ancestors with a single query
Recursion in Databases

• Consider a graph G(V, E). Can you find out all “ancestor” vertices that can reach “x” using Relational Algebra/Calculus?

• NO! – ANCESTOR cannot be defined using a constant-size union of select-project-join queries (conjunctive queries)

• No RA/RC expressions can express ANCESTOR or REACHABILITY (TRANSITIVE CLOSURE) (Aho-Ullman, 1979)

• A limitation of RA/RC in expressing recursive queries
Recursion in Databases

• What can we do to overcome the limitation?

1. Embed SQL in a high level language supporting recursion
   – (-) destroys the high level declarative characteristic of SQL
2. Augment RC with a high level declarative mechanism for recursion
   – Datalog (Chandra-Harel, 1982)

• SQL:1999 (SQL3) and later versions support “linear Datalog”
Brief History of Datalog

• Motivated by Prolog – started back in 80’s – then quiet for a long time

• A long argument in the Database community whether recursion should be supported in query languages
  – “No practical applications of recursive query theory ... have been found to date” — Michael Stonebraker, 1998
    *Readings in Database Systems, 3rd Edition* Stonebraker and Hellerstein, eds.
  – Recent work by Hellerstein et al. on Datalog-extensions to build networking protocols and distributed systems. [Link]
Datalog is resurging!

- Number of papers and tutorials in DB conferences

- Applications in
  - data integration, declarative networking, program analysis, information extraction, network monitoring, security, and cloud computing

- Systems supporting datalog in both academia and industry:
  - Lixto (information extraction)
  - LogicBlox (enterprise decision automation)
  - Semmle (program analysis)
  - BOOM/Dedalus (Berlekey)
  - Coral
  - LDL++
Optional:

1. The datalog chapters in the “Alice Book”
   Foundations of Databases
   Abiteboul-Hull-Vianu
   Available online: http://webdam.inria.fr/Alice/

2. Datalog tutorial
   SIGMOD 2011
   “Datalog and Emerging Applications: An Interactive Tutorial”

Acknowledgement:
Some of the following slides have been borrowed from
slides by Prof. Jun Yang
Recursive Query in SQL
Recursion in SQL

• SQL2 had no recursion

• SQL3 introduces recursion
  – WITH clause
  – Implemented in PostgreSQL (common table expressions)
Ancestor query in SQL3

WITH RECURSIVE Ancestor(anc, desc) AS
    (SELECT parent, child FROM Parent)
     UNION
    (SELECT a1.anc, a2.desc
     FROM Ancestor a1, Ancestor a2
     WHERE a1.desc = a2.anc)

SELECT anc
FROM Ancestor
WHERE desc = 'Bart';
Fixed point of a function

- If $f: T \rightarrow T$ is a function from a type $T$ to itself, a fixed point of $f$ is a value $x$ such that $f(x) = x$

- Example: What is the fixed point of $f(x) = x/2$?  
  - $0$, because $f(0) = 0/2 = 0$
To compute fixed point of a function $f$

- Start with a “seed”: $x \leftarrow x_0$
- Compute $f(x)$
  - If $f(x) = x$, stop; $x$ is fixed point of $f$
  - Otherwise, $x \leftarrow f(x)$; repeat

- Example: compute the fixed point of $f(x) = x/2$
  - With seed 1: 1, 1/2, 1/4, 1/8, 1/16, ... $\rightarrow 0$

️ Doesn’t always work, but happens to work for us!
Fixed point of a query

• A query \( q \) is just a function that maps an input table to an output table, so a fixed point of \( q \) is a table \( T \) such that \( q(T) = T \)

To compute fixed point of \( q \)

• Start with an empty table: \( T \leftarrow \emptyset \)
• Evaluate \( q \) over \( T \)
  – If the result is identical to \( T \), stop; \( T \) is a fixed point
  – Otherwise, let \( T \) be the new result; repeat

Starting from \( \emptyset \) produces the unique minimal fixed point (assuming \( q \) is monotone)
Finding ancestors

- WITH RECURSIVE
  Ancestor(anc, desc) AS
  ((SELECT parent, child FROM Parent)
  UNION
  (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
  — Think of the definition as Ancestor = q(Ancestor)

<table>
<thead>
<tr>
<th>parent</th>
<th>child</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homer</td>
<td>Bart</td>
</tr>
<tr>
<td>Homer</td>
<td>Lisa</td>
</tr>
<tr>
<td>Marge</td>
<td>Bart</td>
</tr>
<tr>
<td>Marge</td>
<td>Lisa</td>
</tr>
<tr>
<td>Abe</td>
<td>Homer</td>
</tr>
<tr>
<td>Ape</td>
<td>Abe</td>
</tr>
</tbody>
</table>

<table>
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<tr>
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<th>desc</th>
</tr>
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</tr>
<tr>
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<td>Homer</td>
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<tr>
<td>Ape</td>
<td>Abe</td>
</tr>
<tr>
<td>Abe</td>
<td>Bart</td>
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<tr>
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<td>Lisa</td>
</tr>
<tr>
<td>Ape</td>
<td>Homer</td>
</tr>
<tr>
<td>Ape</td>
<td>Lisa</td>
</tr>
</tbody>
</table>
Linear recursion

- With linear recursion, a recursive definition can make only one reference to itself
- Non-linear
  - WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent)
     UNION
     (SELECT a1.anc, a2.desc
      FROM Ancestor a1, Ancestor a2
      WHERE a1.desc = a2.anc))
- Linear
  - WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent)
     UNION
     (SELECT anc, child
      FROM Ancestor, Parent
      WHERE desc = parent))
Linear vs. non-linear recursion

• Linear recursion is easier to implement
  – For linear recursion, just keep joining “newly generated” Ancestor rows with Parent
    • Homework: try to figure out why it should work
  – For non-linear recursion, need to join newly generated Ancestor rows with all existing Ancestor rows

• Non-linear recursion may take fewer steps to converge, but perform more work
  – Example: $a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$
  – Linear recursion takes 4 steps
  – Non-linear recursion takes 3 steps
    • More work: e.g., $a \rightarrow d$ has two different derivations
Today

• Finish recursion/datalog + views
• Finish Selinger’s algorithm from query optimization lecture
  – Lecture 12
Announcements

• **No class on Thursday**
  – Happy thanksgiving!
  – No office hours during the break (post on piazza, schedule an appointment)

• **Today we will have the 5th pop-up quiz**
  – Will be posted at the end of the class
  – Will be open for 24 hours

• **There might be a 6th (and last) pop-up quiz next Tuesday**
  – either in class or take home
  – Topic: Query evaluation + optimization
    • Lecture 10, 12, Selinger’s algorithm from today

• **There will be some practice pop-up quizzes during the study break**
  – won’t be graded
  – will be open for 2 days, then the answers will be revealed
Announcements

• **Project presentation in class on Thursday Nov 29**
  – So that everyone knows what you have been working on!
  – And can compare with your progress

• **But you will submit final report 2 days before the final exam**
  – you can keep working on the project
  – we will give you feedback in the next few days
  – there might be a short 15 mins meeting with instructor + TAs if we have questions the week before the final exam
  – final grade of projects will depend on the final outcome/report (not the status in the presentation)
Announcements

• Presentation
  – 14 projects in 75 mins – 4 mins per project!
  – Not everyone has to present (up to you)
    • everyone in a group gets the same grade
  – You present the current status of the project
    • problem, example, your approach, what you plan
  – Best to show plots/ screenshots/ results/ demo!
  – Try to show the most interesting observation/findings in 4 mins!
  – Tell us what you want to do before you submit the final report (if anything)
Mutual recursion example

- Table *Natural* \((n)\) contains 1, 2, ..., 100

- Which numbers are even/odd?
  - An odd number plus 1 is an even number
  - An even number plus 1 is an odd number
  - 1 is an odd number

WITH RECURSIVE Even(n) AS
(SELECT n FROM Natural
 WHERE n = ANY(SELECT n+1 FROM Odd)),
RECURSIVE Odd(n) AS
((SELECT n FROM Natural WHERE n = 1)
 UNION
(SELECT n FROM Natural
 WHERE n = ANY(SELECT n+1 FROM Even)))
Semantics of WITH

- **WITH RECURSIVE** $R_1$ AS $Q_1$, ..., 
  **RECURSIVE** $R_n$ AS $Q_n$

$Q$;
- $Q$ and $Q_1$, ..., $Q_n$ may refer to $R_1$, ..., $R_n$

- **Semantics**
  1. $R_1 \leftarrow \emptyset$, ..., $R_n \leftarrow \emptyset$
  2. Evaluate $Q_1$, ..., $Q_n$ using the current contents of $R_1$, ..., $R_n$:
     $R_1^{new} \leftarrow Q_1$, ..., $R_n^{new} \leftarrow Q_n$
  3. If $R_i^{new} \neq R_i$ for some $i$
     3.1. $R_1 \leftarrow R_1^{new}$, ..., $R_n \leftarrow R_n^{new}$
     3.2. Go to 2.
  4. Compute $Q$ using the current contents of $R_1$, ... $R_n$
     and output the result
Computing mutual recursion

WITH RECURSIVE Even(n) AS
  (SELECT n FROM Natural
   WHERE n = ANY(SELECT n+1 FROM Odd)),
RECURSIVE Odd(n) AS
  ((SELECT n FROM Natural WHERE n = 1)
   UNION
   (SELECT n FROM Natural
    WHERE n = ANY(SELECT n+1 FROM Even)))

• $Even = \emptyset$, $Odd = \emptyset$
• $Even = \emptyset$, $Odd = \{1\}$
• $Even = \{2\}$, $Odd = \{1\}$
• $Even = \{2\}$, $Odd = \{1, 3\}$
• $Even = \{2, 4\}$, $Odd = \{1, 3\}$
• $Even = \{2, 4\}$, $Odd = \{1, 3, 5\}$
• …
Mixing negation with recursion

• If $q$ is non-monotone
  – The fixed-point iteration may flip-flop and never converge
  – There could be multiple minimal fixed points—we wouldn’t know which one to pick as answer!

• Example: popular users ($\text{pop} \geq 0.8$) join either Jessica’s Circle or Tommy’s (but not both)
  – Those not in Jessica’s Circle should be in Tom’s
  – Those not in Tom’s Circle should be in Jessica’s

• \text{WITH RECURSIVE} TommyCircle(uid) AS
  (SELECT uid FROM User WHERE pop $\geq$ 0.8
   AND uid NOT IN (SELECT uid FROM JessicaCircle)),
\text{RECURSIVE} JessicaCircle(uid) AS
  (SELECT uid FROM User WHERE pop $\geq$ 0.8
   AND uid NOT IN (SELECT uid FROM TommyCircle))
Fixed-point iter may not converge

- WITH RECURSIVE TommyCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
  AND uid NOT IN (SELECT uid FROM JessicaCircle)),
RECURSIVE JessicaCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
  AND uid NOT IN (SELECT uid FROM TommyCircle))

<table>
<thead>
<tr>
<th>uid</th>
<th>name</th>
<th>age</th>
<th>pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>142</td>
<td>Bart</td>
<td>10</td>
<td>0.9</td>
</tr>
<tr>
<td>121</td>
<td>Allison</td>
<td>8</td>
<td>0.85</td>
</tr>
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</table>
Multiple minimal fixed points

- WITH RECURSIVE TommyCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
   AND uid NOT IN (SELECT uid FROM JessicaCircle)),
RECURSIVE JessicaCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
   AND uid NOT IN (SELECT uid FROM TommyCircle))

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Problem: What do we answer if someone asks whether 121 belongs to JessicaCircle?
Legal mix of negation and recursion

• Construct a dependency graph
  – One node for each table defined in WITH
  – A directed edge $R \rightarrow S$ if $R$ is defined in terms of $S$
  – Label the directed edge “—” if the query defining $R$ is not monotone with respect to $S$

• Legal SQL3 recursion: no cycle with a “—” edge
  – Called stratified negation

• Bad mix: a cycle with at least one edge labeled “—”
Stratified negation example

- Find pairs of persons with no common ancestors

WITH RECURSIVE Ancestor(anc, desc) AS
  ((SELECT parent, child FROM Parent) UNION
  (SELECT a1.anc, a2.desc FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc)),

Person(person) AS
  ((SELECT parent FROM Parent) UNION
  (SELECT child FROM Parent)),

NoCommonAnc(person1, person2) AS
  ((SELECT p1.person, p2.person FROM Person p1, Person p2
  WHERE p1.person <> p2.person)
  EXCEPT
  (SELECT a1.desc, a2.desc FROM Ancestor a1, Ancestor a2
  WHERE a1.anc = a2.anc))

SELECT * FROM NoCommonAnc;
Evaluating stratified negation

- The stratum of a node $R$ is the maximum number of “$-$” edges on any path from $R$ in the dependency graph
  - $Ancestor$: stratum 0
  - $Person$: stratum 0
  - $NoCommonAnc$: stratum 1

- Evaluation strategy
  - Compute tables lowest-stratum first
  - For each stratum, use fixed-point iteration on all nodes in that stratum
    - Stratum 0: $Ancestor$ and $Person$
    - Stratum 1: $NoCommonAnc$

♀ Intuitively, there is no negation within each stratum
Practice Datalog on whiteboard

• Write Datalog program for reachability:
  – x can reach y
  – start with $E(u, v) : \text{an edge exists from } u \text{ to } v$

• $E(u, v, c)$: an edge exists from $u$ to $v$ of color “c”
  – e.g. $E(1, 2, \text{blue}), E(2, 3, \text{red}), \ldots$.

• Find node pairs $x, y$ such that $x$ can reach $y$ by a blue path
Summary so far

- SQL3 WITH recursive queries
- Solution to a recursive query (with no negation): unique minimal fixed point
- Computing unique minimal fixed point: fixed-point iteration starting from $\emptyset$
- Mixing negation and recursion is tricky
  - Illegal mix: fixed-point iteration may not converge; there may be multiple minimal fixed points
  - Legal mix: stratified negation (compute by fixed-point iteration stratum by stratum)
- Another language for recursion: Datalog
Datalog
Datalog: Another query language for recursion

• Ancestor(x, y) :- Parent(x, y)
• Ancestor(x, y):- Parent(x, z), Ancestor(z, y)

• Like logic programming
• Multiple rules
• Same “head” = union
• “,” = AND

• Same semantics that we discussed so far
Recall our drinker example in RC (Lecture 4)

Find drinkers that frequent some bar that serves some beer they like.

\[ Q(x) = \exists y. \exists z. \text{Frequents}(x, y) \land \text{Serves}(y, z) \land \text{Likes}(x, z) \]
Write it as a Datalog Rule

Find drinkers that frequent some bar that serves some beer they like.

RC:
\[ Q(x) = \exists y. \exists z. \text{Frequents}(x,y) \land \text{Serves}(y,z) \land \text{Likes}(x,z) \]

Datalog:
\[ Q(x) :- \text{Frequents}(x,y), \text{Serves}(y,z), \text{Likes}(x,z) \]
Write it as a Datalog Rule

Find drinkers that frequent some bar that serves some beer they like.

RC:
Q(x) = ∃y. ∃z. Frequents(x, y) ∧ Serves(y,z) ∧ Likes(x,z)

Datalog:
Q(x) :- Frequents(x, y), Serves(y,z), Likes(x,z)

• Quick differences:
  – Uses “:-” not =
  – no need for ∃ (assumed by default)
  – Use “,” on the right hand side (RHS)
  – Anything on RHS the of :- is assumed to be combined with ∧ by default
  – ∀, ⇒, not allowed – they need to use negation ¬
  – Standard “Datalog” does not allow negation
  – Negation allowed in datalog with negation

• How to specify disjunction (OR / ∨)?
Example: OR in Datalog

Find drinkers that (a) either frequent some bar that serves some beer they like, (b) or like beer “BestBeer”

RC:
Q(x) = [∃y. ∃z. Frequents(x, y) ∧ Serves(y,z) ∧ Likes(x,z)] ∨ [Likes(x, “BestBeer”)]

Datalog:
Q(x) :- Frequents(x, y), Serves(y,z), Likes(x,z)
Q(x) :- Likes(x, “BestBeer”)
Example: OR in Datalog

Find drinkers that (a) either frequent some bar that serves some beer they like, (b) or like beer “BestBeer”, (c) or, frequent bars that “Joe” frequents

RC:
Q(x) = \[\exists y. \exists z. \text{Frequents}(x, y) \land \text{Serves}(y,z) \land \text{Likes}(x,z)] \lor \[\text{Likes}(x, "BestBeer")\] \\
\lor \[\exists w \text{Frequents}(x, w) \land \text{Frequents}("Joe", w)\]

Datalog:
JoeFrequents(w) :- \text{Frequents}("Joe", w) \\
Q(x) :- \text{Frequents}(x, y), \text{Serves}(y,z), \text{Likes}(x,z) \\
Q(x) :- \text{Likes}(x, "BestBeer") \\
Q(x) :- \text{Frequents}(x, w), \text{JoeFrequents}(w)

• To specify “OR”, write multiple rules with the same “Head”
• Next: terminology for Datalog
Each rule is of the form \textbf{Head} :- \textbf{Body}

Each variable in the head of each rule must appear in the body of the rule

JoeFrequents(w) :- Frequents("Joe", w)
Q(x) :- Frequents(x, y), Serves(y, z), Likes(x, z)
Q(x) :- Likes(x, "BestBeer")
Q(x) :- Frequents(x, w), JoeFrequents(w)
Termination of a Datalog Program

Q. A Datalog program always terminates – why?
Unsafe/Safe Datalog Rules

Find drinkers who like beer “BestBeer”

Find drinkers who DO NOT like beer “BestBeer”

Q(x) :- Likes(x, “BestBeer”)

Q(x) :- ¬Likes(x, “BestBeer”)

• What is the problem with this rule?
• What should this rule return?
  – names of all drinkers in the world?
  – names of all drinkers in the USA?
  – names of all drinkers in Durham?

Another Problem with Negation in Datalog Rules
Find drinkers who like beer “BestBeer”

Find drinkers who DO NOT like beer “BestBeer”

- What is the problem with this rule?
  - Dependent on “domain” of drinkers
    - domain-dependent
    - infinite answers possible too..
      - keep generating “names”
    - Unsafe rule

Domain-dependency is bad

Another Problem with Negation in Datalog Rules
Safe Datalog Rules

Find drinkers who like beer “BestBeer”

\[ Q(x) : \neg Likes(x, “BestBeer”) \]

Find drinkers who DO NOT like beer “BestBeer”

\[ Q(x) : Likes(x, “BestBeer”) \]

• Solution:

• Restrict to “active domain” of drinkers from the input \textit{Likes} (or \textit{Frequents}) relation
  – “domain-independence” – same finite answer always

• Becomes a “safe rule”

\[ Q(x) : Likes(x, y), \neg Likes(x, “BestBeer”) \]
Views

- A view is like a “virtual” table
  - Defined by a query, which describes how to compute the view contents on the fly
  - DBMS stores the view definition query instead of view contents
  - Can be used in queries just like a regular table
Creating and dropping views

• Example: members of Jessica’s Circle
  – CREATE VIEW JessicaCircle AS
    SELECT * FROM User
    WHERE uid IN (SELECT uid FROM Member
                 WHERE gid = 'jes');

  – Tables used in defining a view are called “base tables”
    • User and Member above

• To drop a view
  – DROP VIEW JessicaCircle;
Using views in queries

- Example: find the average popularity of members in Jessica’s Circle

  - SELECT AVG(pop) FROM JessicaCircle;

  - To process the query, replace the reference to the view by its definition

  - SELECT AVG(pop) FROM (SELECT * FROM User
      WHERE uid IN
      (SELECT uid FROM Member
       WHERE gid = 'jes'))
    AS JessicaCircle;
Why use views?

• To hide data from users
• To hide complexity from users

• Logical data independence
  – If applications deal with views, we can change the underlying schema without affecting applications

• To provide a uniform interface for different implementations or sources

☞ Real database applications use tons of views
Selinger’s algorithm for Lecture 12
Task 4: Efficiently searching the plan space

Use dynamic-programming based Selinger’s algorithm
Heuristics for pruning plan space

- Apply predicates as early as possible
- Avoid plans with cross products
- Only left-deep join trees
Join Trees

Query:  \[ R1 \Join R2 \Join R3 \Join R4 \Join R5 \]

- Several possible structure of the trees
- Each tree can have \( n! \) permutations of relations on leaves

(logical plan space)

(physical plan space)

- Different implementation and scanning of intermediate operators for each logical plan
Selinger Algorithm

• Dynamic Programming based
• Dynamic Programming:
  – General algorithmic paradigm
  – Exploits “principle of optimality”
    • Useful reading: Chapter 16, Introduction to Algorithms, Cormen, Leiserson, Rivest
• Considers the search space of left-deep join trees
  – reduces search space (only one structure)
  – but still n! permutations
  – interacts well with join algos (esp. NLJ)
  – e.g. might not need to write tuples to disk if enough memory
Principle of Optimality

Optimal for “whole” made up from optimal for “parts”
Principle of Optimality

Query:  $R1 \bowtie R2 \bowtie R3 \bowtie R4 \bowtie R5$

Suppose, this is an Optimal Plan for joining $R1...R5$: 
Principle of Optimality

Query: \( R1 \bowtie R2 \bowtie R3 \bowtie R4 \bowtie R5 \)

Then, what can you say about this sub-plan?

This has to be the optimal plan for joining \( R3, R2, R4, R1 \)

Suppose, this is an Optimal Plan for joining \( R1 \ldots R5 \):
Principle of Optimality

Query: \[ R1 \Join R2 \Join R3 \Join R4 \Join R5 \]

Suppose, this is an Optimal Plan for joining \( R1 \ldots R5 \):

Then, what can you say about this sub-plan?

We are using the associativity and commutativity of joins:

\[(R \Join S) \Join T = R \Join (S \Join T)\]
\[R \Join S = S \Join R\]

This has to be the optimal plan for joining \( R3, R2, R4 \)
Exploiting Principle of Optimality

Query: \( R_1 \bowtie R_2 \bowtie \ldots \bowtie R_n \)

Both are giving the same result
\( R_2 \bowtie R_3 \bowtie R_1 = R_3 \bowtie R_1 \bowtie R_2 \)

Optimal for joining \( R_1, R_2, R_3 \)

Sub-Optimal for joining \( R_1, R_2, R_3 \)
Exploiting Principle of Optimality

Suppose you chose the sub-optimal one

Leads to sub-Optimal for joining R1,…,Rn

A sub-optimal sub-plan cannot lead to an optimal plan
Notation

OPT ( \{ R_1, R_2, R_3 \} ):

Cost of optimal plan to join R_1, R_2, R_3

T ( \{ R_1, R_2, R_3 \} ):

Number of tuples in \( R_1 \bowtie R_2 \bowtie R_3 \)
Simple Cost Model

\[
\text{Cost } (R \bowtie S) = T(R) + T(S)
\]

All other operators have 0 cost

Note: The simple cost model used for illustration only, it is not used in practice
Cost Model Example

\[
\begin{align*}
\text{Total Cost: } & T(R) + T(S) + T(T) + T(X) \\
\end{align*}
\]
Selinger Algorithm:

\[ \text{OPT ( \{ R1, R2, R3 \} ) :} \]

\[
\begin{align*}
\text{Min} & \quad \text{OPT ( \{ R1, R2 \} )} + T ( \{ R1, R2 \} ) + T(R3) \\
& \quad \text{OPT ( \{ R2, R3 \} )} + T ( \{ R2, R3 \} ) + T(R1) \\
& \quad \text{OPT ( \{ R1, R3 \} )} + T ( \{ R1, R3 \} ) + T(R2)
\end{align*}
\]

\textbf{Note: Valid only for the simple cost model}
Selinger Algorithm:

Query:  \( R1 \bowtie R2 \bowtie R3 \bowtie R4 \)

\[\{ R1, R2, R3, R4 \}\]
\[\{ R1, R2, R3 \}\]  \[\{ R1, R2, R4 \}\]  \[\{ R1, R3, R4 \}\]  \[\{ R2, R3, R4 \}\]
\[\{ R1, R2 \}\]  \[\{ R1, R3 \}\]  \[\{ R1, R4 \}\]  \[\{ R2, R3 \}\]  \[\{ R2, R4 \}\]  \[\{ R3, R4 \}\]
\[\{ R1 \}\]  \[\{ R2 \}\]  \[\{ R3 \}\]  \[\{ R4 \}\]

Progress of algorithm
Selinger Algorithm:

Query: $R1 \bowtie R2 \bowtie R3 \bowtie R4$

Progress of algorithm
Selinger Algorithm:

Query: \( R1 \bowtie R2 \bowtie R3 \bowtie R4 \)

*Progress of algorithm*

- e.g. All possible permutations of \( R1, R3, R4 \) have been considered after \( \text{OPT}\{R1, R3, R4\} \) has been computed.
Selinger Algorithm:

Query: \( R1 \bowtie R2 \bowtie R3 \bowtie R4 \)

Q. How to optimally compute join of \( \{R1, R2, R3, R4\} \)?

Ans: First optimally join \( \{R1, R3, R4\} \) then join with \( R2 \) as inner.
Selinger Algorithm:

Query: \( R_1 \bowtie R_2 \bowtie R_3 \bowtie R_4 \)

Q. How to optimally compute join of \( \{R_1, R_3, R_4\} \)?

Ans: First optimally join \( \{R_1, R_3\} \), then join with \( R_4 \) as inner.
Selinger Algorithm:

Query:  \( R1 \bowtie R2 \bowtie R3 \bowtie R4 \)

Q. How to optimally compute join of \{R1, R3\}? 

Ans: First optimally join \{R3\}, then join with \( R1 \) as inner.
Selinger Algorithm:

Query:  \( R1 \bowtie R2 \bowtie R3 \bowtie R4 \)

Q. How to optimally compute join of \{R3\}?

Ans: Single relation – so optimally scan R3.
Selinger Algorithm:

Final optimal plan:

Query:  $R1 \bowtie R2 \bowtie R3 \bowtie R4$

NOTE: There is a one-one correspondence between the permutation (R3, R1, R4, R2) and the above left deep plan
Selinger Algorithm:

Query: $R_1 \bowtie R_2 \bowtie R_3 \bowtie R_4$

NOTE: (*VERY IMPORTANT*)

- This is *NOT* done by top-down recursive calls.
- This is done BOTTOM-UP computing the optimal cost of *all* nodes in this lattice only once (dynamic programming).

Progress of algorithm

{ $R_1, R_2, R_3, R_4$ }
{ $R_1, R_2, R_3$ }  { $R_1, R_2, R_4$ }  { $R_1, R_3, R_4$ }  { $R_2, R_3, R_4$ }
{ $R_1, R_2$ }  { $R_1, R_3$ }  { $R_1, R_4$ }  { $R_2, R_3$ }  { $R_2, R_4$ }  { $R_3, R_4$ }
{ $R_1$ }  { $R_2$ }  { $R_3$ }  { $R_4$ }
More on Query Optimizations

• See the survey (on course website):
  “An Overview of Query Optimization in Relational Systems” by Surajit Chaudhuri

• Covers other aspects like
  – Pushing group by before joins
  – Merging views and nested queries
  – “Semi-join”-like techniques for multi-block queries
    • covered in distributed databases
  – Statistics and optimizations
  – Starbust and Volcano/Cascade architecture, etc