CompSci 516
Database Systems

Lecture 7-8
Index
(B+-Tree and Hash)

Instructor: Sudeepa Roy

Announcements

- HW1 and project proposal deadlines next week:
  – Due on 09/27 (Thurs), 11:55 pm, no late days
  – HW1 submission on gradescope (code on piazza)
  – Proposal submission on sakai (one per group)
  – Project ideas on sakai
- Do not forget to start homeworks early!
  – Especially for the next two HW

Reading Material

- [RG]
  – Storage: Chapters 8.1, 8.2, 8.4, 9.4-9.7
  – Index: 8.3, 8.5
  – Tree-based index: Chapter 10.1-10.7
  – Hash-based index: Chapter 11

Additional reading
- [GUW]
  – Chapters 8.3, 14.1-14.4

Acknowledgement:
The following slides have been created adapting the instructor material of the [RG] book provided by the authors Dr. Ramakrishnan and Dr. Gehrke.

Recap

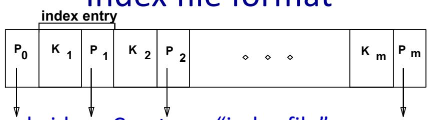
- Storage :
  – Files -> Records -> Fields
  – Fixed and variable length
- Index
  – Search key k -> Data entry k* -> Record
  – Alternative 1/2/3 for k*
  – Primary/secondary, clustered/unclustered
- Today
  – B+ tree index
  – Hash based index

Tree-based Index and B+-Tree

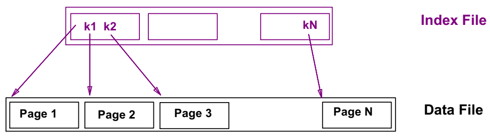
Range Searches

- *``Find all students with gpa > 3.0''*
  – If data is in sorted file, do binary search to find first such student, then scan to find others.
  – Cost of binary search can be quite high.

Index file format



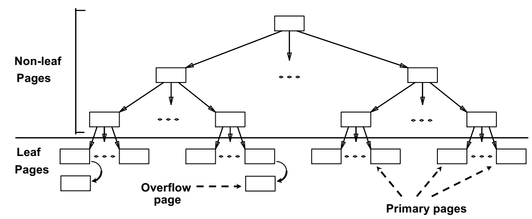
- Simple idea: Create an "index file"
  – <first-key-on-page, pointer-to-page>, sorted on keys



Can do binary search on (smaller) index file
but may still be expensive: apply this idea repeatedly

Indexed Sequential Access Method (ISAM)

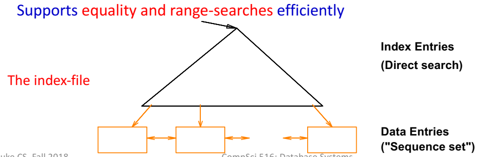
- Leaf-pages contain data entry – also some overflow pages
- DBMS organizes layout of the index – a static structure
- If a number of inserts to the same leaf, a long overflow chain can be created
  – affects the performance



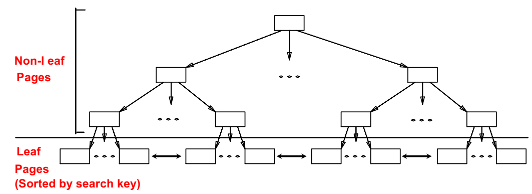
Leaf pages contain data entries.

B+ Tree

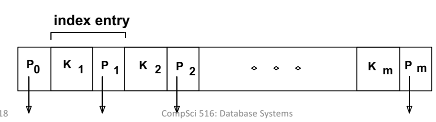
- Most Widely Used Index
  – a dynamic structure
- Insert/delete at $\log_F N$ cost = height of the tree (cost = I/O)
  – F = fanout, N = no. of leaf pages
  – tree is maintained height-balanced
- Minimum 50% occupancy
  – Each node contains **d** <= m <= 2**d** entries
  – Root contains 1 <= m <= 2d entries
  – The parameter **d** is called the order of the tree
- Supports equality and range-searches efficiently



B+ Tree Indexes

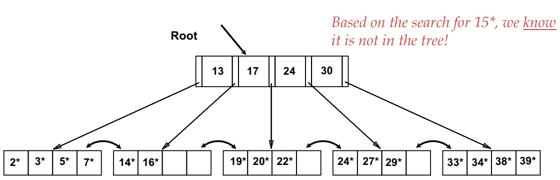


- Leaf pages contain data entries, and are chained (prev & next)
- Non-leaf pages have index entries; only used to direct searches:

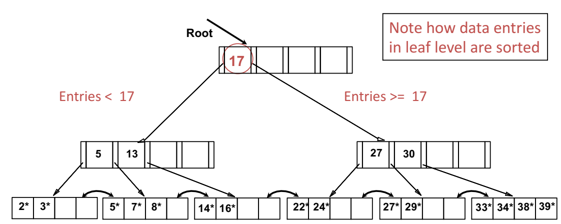


Example B+ Tree

- Search begins at root, and key comparisons direct it to a leaf
- Search for 5*, 15*, all data entries >= 24* ...



Example B+ Tree



- Find
  – 28*?
  – 29*?
  – All > 15* and < 30*

B+ Trees in Practice

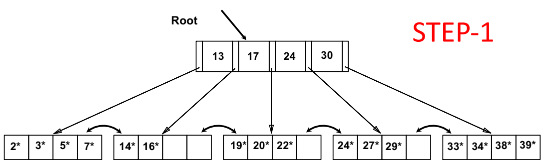
- Typical order: d = 100. Typical fill-factor: 67%
  - average fanout F = 133
- Typical capacities:
  - Height 4: 133^4 = 312,900,700 records
  - Height 3: 133^3 = 2,352,637 records
- Can often hold top levels in buffer pool:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes

Inserting a Data Entry into a B+ Tree

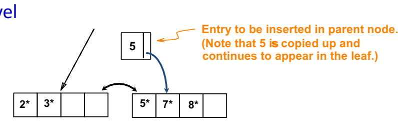
See this slide later, First, see examples on the next few slides

- Find correct leaf L
- Put data entry onto L
  - If L has enough space, done
  - Else, must split L
    - into L and a new node L2
    - Redistribute entries evenly, copy up middle key.
    - Insert index entry pointing to L2 into parent of L.
- This can happen recursively
  - To split index node, redistribute entries evenly, but push up middle key
    - **Contrast with leaf splits**
- Splits "grow" tree; root split increases height.
  - Tree growth: gets wider or one level taller at top.

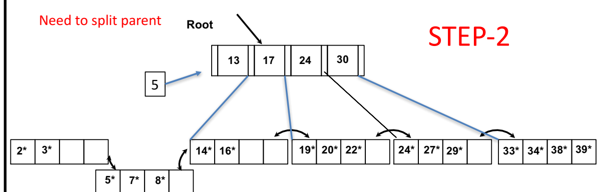
Inserting 8* into Example B+ Tree



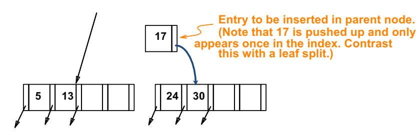
- Copy-up: 5 appears in leaf and the level above
- Observe how minimum occupancy is guaranteed



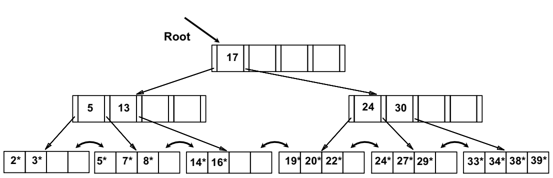
Inserting 8* into Example B+ Tree



- Note difference between copy-up and push-up
- What is the reason for this difference?
- All data entries must appear as leaves
  - (for easy range search)
- no such requirement for indexes
  - (so avoid redundancy)



Example B+ Tree After Inserting 8*



- Notice that root was split, leading to increase in height.
- In this example, we can avoid split by re-distributing entries (insert 8 to the 2nd leaf node from left and copy it up instead of 13)
  - however, this is usually not done in practice – since need to access 1-2 extra pages always (for two siblings), and average occupancy may remain unaffected as the file grows

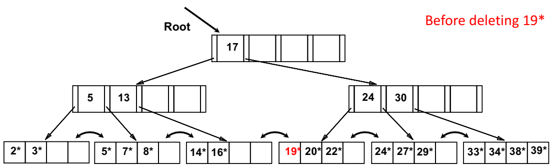
Deleting a Data Entry from a B+ Tree

Each non-root node contains d <= m <= 2d entries

- Start at root, find leaf L where entry belongs
- Remove the entry
  - If L is at least half-full, done!
  - If L has only **d-1** entries,
    - Try to re-distribute, borrowing from sibling (adjacent node with same parent as L)
    - If re-distribution fails, merge L and sibling
- If merge occurred, must delete entry (pointing to L or sibling) from parent of L
- Merge could propagate to root, decreasing height

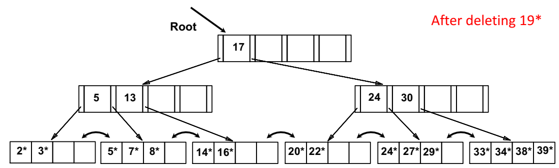
See this slide later, First, see examples on the next few slides

Example Tree: Delete 19*

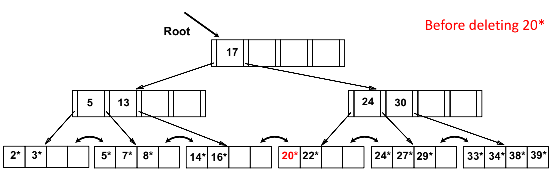


- We had inserted 8*
- Now delete 19*
- Easy

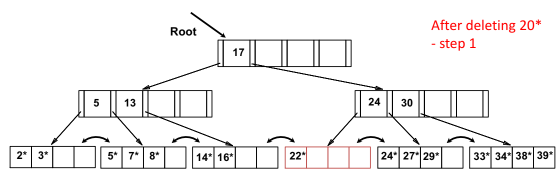
Example Tree: Delete 19*



Example Tree: Delete 20*

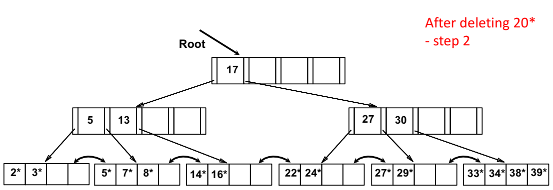


Example Tree: Delete 20*



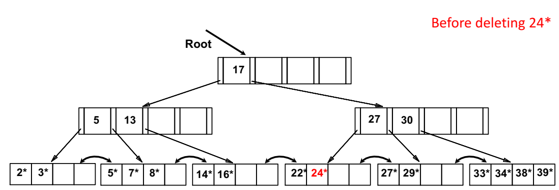
- < 2 entries in leaf-node
- Redistribute

Example Tree: Delete 20*



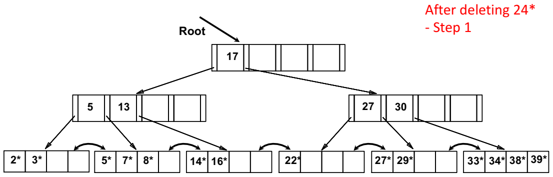
- Notice how middle key is copied up

Example Tree: ... And Then Delete 24*



Example Tree: ... And Then Delete 24*

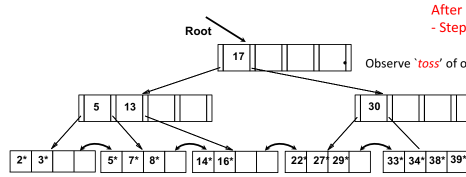
After deleting 24*
- Step 1



- Once again, imbalance at leaf
- Can we borrow from sibling(s)?
- No – d-1 and d entries (d = 2)
- Need to merge

Example Tree: ... And Then Delete 24*

After deleting 24*
- Step 2

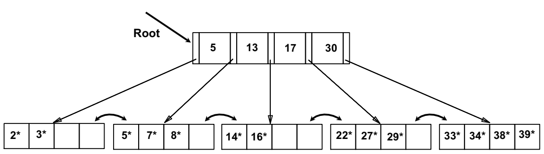


Observe `*toss*' of old index entry 27

- Imbalance at parent
- Merge again
- But need to "pull down" root index entry

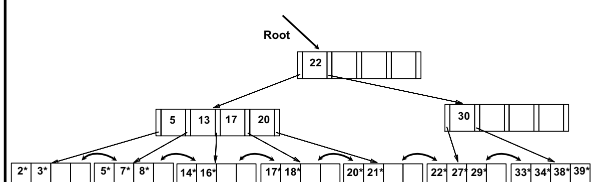
because, three index 5, 13, 30 but five pointers to leaves

Final Example Tree



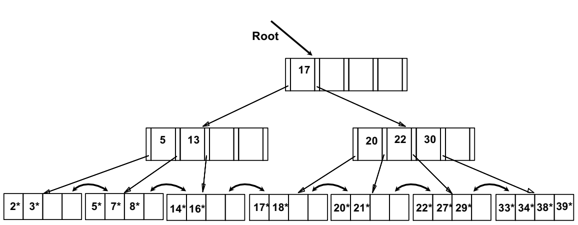
Example of Non-leaf Re-distribution

- An intermediate tree is shown
- In contrast to previous example, can re-distribute entry from left child of root to right child



After Re-distribution

- Intuitively, entries are re-distributed by `pushing through' the splitting entry in the parent node.
  - It suffices to re-distribute index entry with key 20; we've re-distributed 17 as well for illustration.



Duplicates

- First Option:
  - The basic search algorithm assumes that all entries with the same key value resides on the same leaf page
  - If they do not fit, use overflow pages (like ISAM)
- Second Option:
  - Several leaf pages can contain entries with a given key value
  - Search for the left most entry with a key value, and follow the leaf-sequence pointers
  - Need modification in the search algorithm
- if k* = <k, rid>, several entries have to be searched
  - Or include rid in k – becomes unique index, no duplicate
  - If k* = <k, rid-list>, some solution, but if the list is long, again a single entry can span multiple pages

A Note on `Order'

- Order (d)
  - denotes minimum occupancy
- replaced by physical space criterion in practice (`at least half-full')
  - Index pages can typically hold many more entries than leaf pages
  - Variable sized records and search keys mean different nodes will contain different numbers of entries.
  - Even with fixed length fields, multiple records with the same search key value (duplicates) can lead to variable-sized data entries (if we use Alternative (3))

Summary

- Tree-structured indexes are ideal for range-searches, also good for equality searches
- ISAM is a static structure
  - Only leaf pages modified; overflow pages needed
  - Overflow chains can degrade performance unless size of data set and data distribution stay constant
- B+ tree is a dynamic structure
  - Inserts/deletes leave tree height-balanced; $\log_F N$ cost
  - High fanout (**F**) means depth rarely more than 3 or 4
  - Almost always better than maintaining a sorted file
  - Most widely used index in database management systems because of its versatility.
  - One of the most optimized components of a DBMS
- Next: Hash-based index

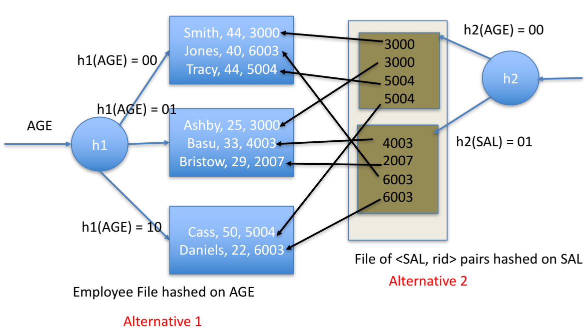
Hash-based Index

Hash-Based Indexes

- Records are grouped into buckets
  - Bucket = primary page plus zero or more overflow pages
- Hashing function **h**:
  - **h**(r) = bucket in which (data entry for) record r belongs
  - **h** looks at the search key fields of r
  - No need for “index entries” in this scheme

Example: Hash-based index

Index organized file hashed on AGE, with Auxiliary index on SAL

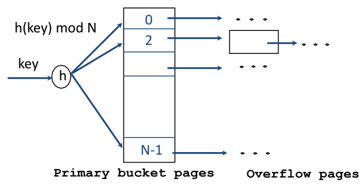


Introduction

- Hash-based indexes are best for equality selections
  - Find all records with name = “Joe”
  - Cannot support range searches
  - But useful in implementing relational operators like join (later)
- Static and dynamic hashing techniques exist
  - trade-offs similar to ISAM vs. B+ trees

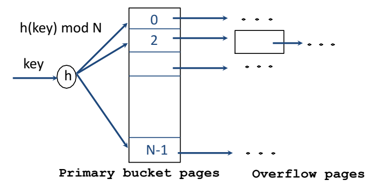
Static Hashing

- Pages containing data = a collection of buckets
  - each bucket has one primary page, also possibly overflow pages
  - buckets contain data entries k*



Static Hashing

- # primary pages fixed
  - allocated sequentially, never de-allocated, overflow pages if needed.
- **h**(k) mod N = bucket to which data entry with key k belongs
  - N = # of buckets



Static Hashing

- Hash function works on search key field of record r
  - Must distribute values over range 0 ... N-1
  - h(key) = (a * key + b) usually works well
    - bucket = h(key) mod N
  - a and b are constants – chosen to tune h
- Advantage:
  - #buckets known – pages can be allocated sequentially
  - search needs 1 I/O (if no overflow page)
  - insert/delete needs 2 I/O (if no overflow page) (why 2?)
- Disadvantage:
  - Long overflow chains can develop if file grows and degrade performance
  - Or waste of space if file shrinks
- Solutions:
  - keep some pages say 80% full initially
  - Periodically rehash if overflow pages (can be expensive)
  - or use Dynamic Hashing

Dynamic Hashing Techniques

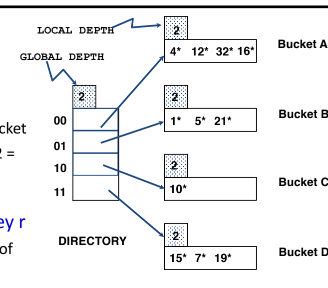
- Extendible Hashing
- Linear Hashing

Extendible Hashing

- Consider static hashing
- Bucket (primary page) becomes full
- Why not re-organize file by doubling # of buckets?
  - Reading and writing (double #pages) all pages is expensive
- Idea: Use directory of pointers to buckets
  - double # of buckets by doubling the directory, splitting just the bucket that overflowed
  - Directory much smaller than file, so doubling it is much cheaper
  - Only one page of data entries is split
  - No overflow page (new bucket, no new overflow page)
  - Trick lies in how hash function is adjusted

Example

- Directory is array of size 4
  - each element points to a bucket
  - #bits to represent = log 4 = 2 = global depth
- To find bucket for search key r
  - take last global depth # bits of **h**(r)
  - assume h(r) = r
  - If **h**(r) = 5 = binary 101
  - it is in bucket pointed to by 01



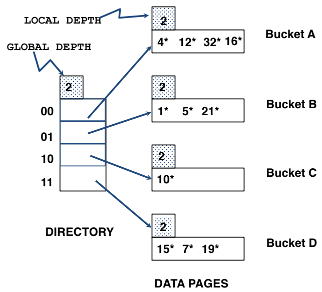
Example

Insert:

- If bucket is full, split it
- allocate new page
- re-distribute

Suppose inserting 13*

- binary = 1101
- bucket 01
- Has space, insert



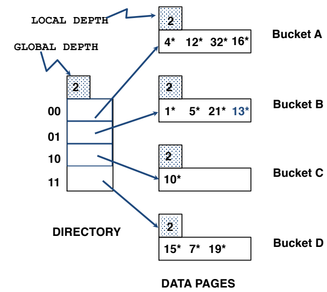
Example

Insert:

- If bucket is full, split it
- allocate new page
- re-distribute

Suppose inserting 20*

- binary = 10100
- bucket 00
- Already full
- To split, consider last three bits of 10100
- Last two bits the same 00 – the data entry will belong to one of these buckets
- Third bit to distinguish them

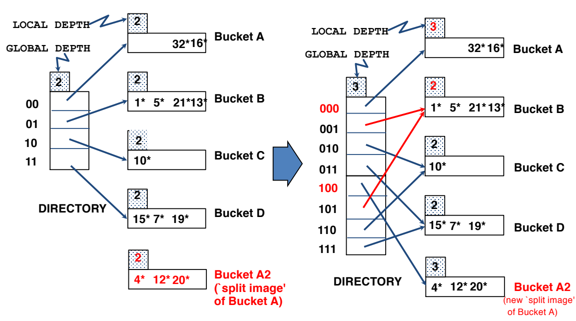


Example

Global depth: Max # of bits needed to tell which bucket an entry belongs to

Local depth: # of bits used to determine if an entry belongs to this bucket

- also denotes whether a directory doubling is needed while splitting
- no directory doubling needed when 9* = 1001 is inserted (LD< GD)



When does bucket split cause directory doubling?

- Before insert, local depth of bucket = global depth
- Insert causes local depth to become > global depth
- directory is doubled by copying it over and `fixing' pointer to split image page

Comments on Extendible Hashing

- If directory fits in memory, equality search answered with one disk access (to access the bucket); else two.
  - 100MB file, 100 bytes/rec, 4KB page size, contains 10^6 records (as data entries) and 25,000 directory elements; chances are high that directory will fit in memory.
  - Directory grows in spurts, and, if the distribution *of hash values* is skewed, directory can grow large
  - Multiple entries with same hash value cause problems
- Delete:
  - If removal of data entry makes bucket empty, can be merged with `split image'
  - If each directory element points to same bucket as its split image, can halve directory.

Linear Hashing

- This is another dynamic hashing scheme
  - an alternative to Extendible Hashing
- LH handles the problem of long overflow chains
  - without using a directory
  - handles duplicates and collisions
  - very flexible w.r.t. timing of bucket splits

Linear Hashing: Basic Idea

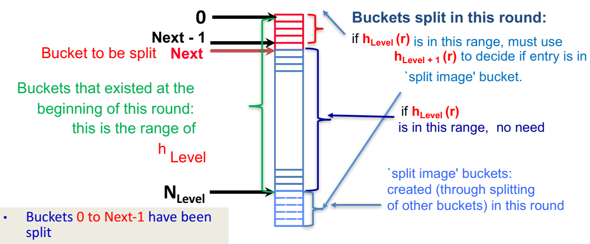
- Use a family of hash functions h_0, h_1, h_2, ...
  - $h_i(key) = h(key) \bmod (2^i N)$
  - N = initial # buckets
  - h is some hash function (range is not 0 to N-1)
  - If $N = 2^{d_0}$, for some d_0, h_i consists of applying h and looking at the last d_i bits, where $d_i = d_0 + i$
    - Note: $h_i(key) = h(key) \bmod (2^{d_0+i})$
  - h_{i+1} doubles the range of h_i
    - if h_i maps to M buckets, h_{i+1} maps to 2M buckets
    - similar to directory doubling
  - Suppose N = 32, d_0 = 5
    - h_0 = h mod 32 (last 5 bits)
    - h_1 = h mod 64 (last 6 bits)
    - h_2 = h mod 128 (last 7 bits) etc.

Linear Hashing: Rounds

- Directory avoided in LH by using overflow pages, and choosing bucket to split round-robin
- During round Level, only h_{Level} and $h_{Level+1}$ are in use
- The buckets from start to last are split sequentially
  - this doubles the no. of buckets
- Therefore, at any point in a round, we have
  - buckets that have been split
  - buckets that are yet to be split
  - buckets created by splits in this round

Overview of LH File

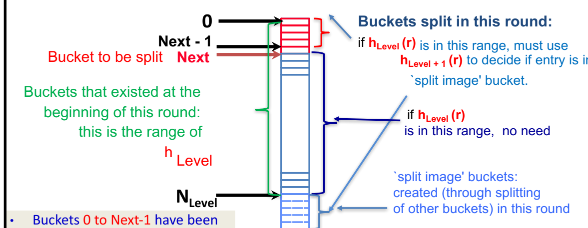
- In the middle of a round Level – originally 0 to N_{Level}



- Buckets 0 to Next-1 have been split
- Next to N_{Level} yet to be split
- Round ends when all N_R initial (for round R) buckets are split

Overview of LH File

- In the middle of a round Level – originally 0 to N_{Level}



- Buckets 0 to Next-1 have been split
- Next to N_{Level} yet to be split
- Round ends when all N_R initial (for round R) buckets are split

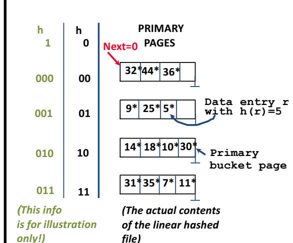
- **Search:** To find bucket for data entry r, find $h_{Level}(r)$:
- If $h_{Level}(r)$ in range `Next to N_{Level}', r belongs here.
- Else, r could belong to bucket $h_{Level}(r)$ or $h_{Level}(r)+N_R$
- Apply $h_{Level+1}(r)$ to find out

Linear Hashing: Insert

- Insert: Find bucket by applying h_{Level} / $h_{Level+1}$:
  - If bucket to insert into is full:
    1. Add overflow page and insert data entry
    2. Split Next bucket and increment Next
- Note: We are going to assume that a split is `triggered' whenever an insert causes the creation of an overflow page, but in general, we could impose additional conditions for better space utilization ([RG], p.380)

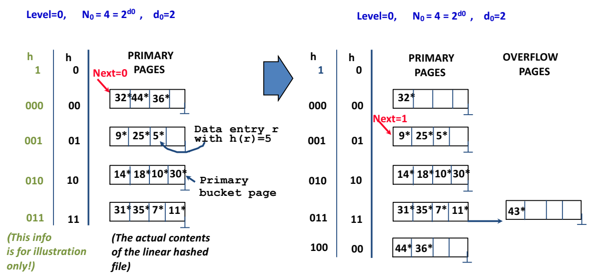
Example of Linear Hashing

Level=0, $N_0 = 4 = 2^{d_0}$, d_0=2



- Insert 43* = 101011
- $h_0(43)$ = 11
- Full
- Insert in an overflow page
- Need a split at Next (=0)
- Entries in 00 is distributed to 000 and 100

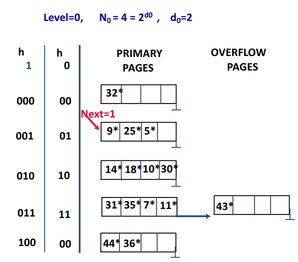
Example of Linear Hashing



- Next is incremented after split
- Note the difference between overflow page of 11 and split image of 00 (000 and 100)

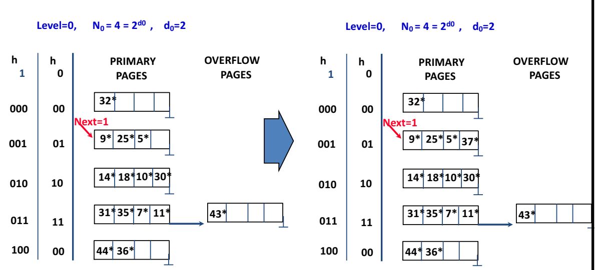
Example of Linear Hashing

- Search for 18* = 10010
  - between Next (=1) and 4
  - this bucket has not been split
    - 18 should be here
- Search for 32* = 100000 or 44* = 101100
- Between 0 and Next-1
  - Need h_1
- Not all insertion triggers split
  - Insert 37* = 100101
  - Has space
- Splitting at Next?
  - No overflow bucket needed
  - Just copy at the image/original
- Next = N_{level}-1 and a split?
  - Start a new round
  - Increment Level
  - Next reset to 0



Example of Linear Hashing

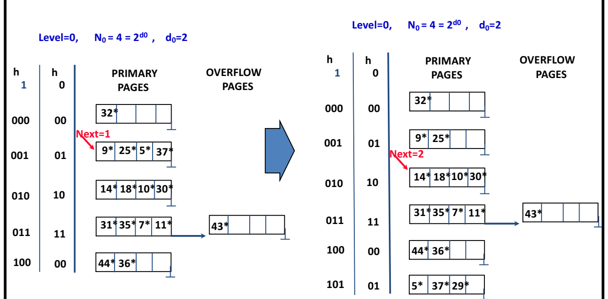
- Not all insertion triggers split
- Insert 37* = 100101
  - Has space



Example of Linear Hashing

- Splitting at Next?
  - No overflow bucket needed
  - Just copy at the image/original

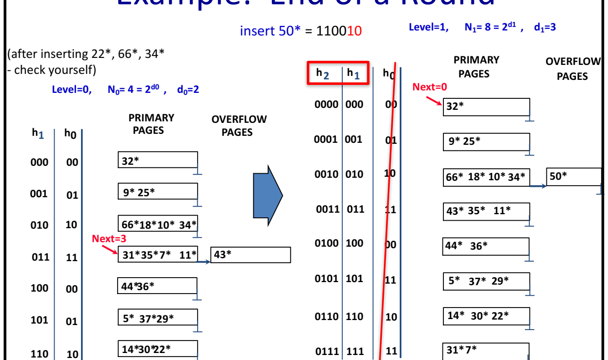
insert 29* = 11101



Example: End of a Round

insert 50* = 110010

(after inserting 22*, 66*, 34*
- check yourself)



LH vs. EH

- They are very similar
  - h_i to h_{i+1} is like doubling the directory
  - LH: avoid the explicit directory, clever choice of split
  - EH: always split – higher bucket occupancy
- Uniform distribution: LH has lower average cost
  - No directory level
- Skewed distribution
  - Many empty/nearly empty buckets in LH
  - EH may be better

Summary

- Hash-based indexes: best for equality searches, cannot support range searches.
- Static Hashing can lead to long overflow chains.
- Extendible Hashing avoids overflow pages by splitting a full bucket when a new data entry is to be added to it
  - Duplicates may still require overflow pages
  - Directory to keep track of buckets, doubles periodically
  - Can get large with skewed data; additional I/O if this does not fit in main memory

Summary

- Linear Hashing avoids directory by splitting buckets round-robin, and using overflow pages
  - Overflow pages not likely to be long
  - Duplicates handled easily
- For hash-based indexes, a skewed data distribution is one in which the *hash values* of data entries are not uniformly distributed
  - bad