1 Combinatorics of Joint Distributions (10 points)

Suppose you have an event space defined by $n$ random variables, each of which can take on $k$ different values, and that you are provided with a joint probability distribution (in the form of a table) over all atomic events in the space.

1. How many atomic events are there in the event space? (2 points)

2. How many additions and subtractions are required to compute the marginal probability distribution of a single variable? (4 points)

3. How many additions and subtractions are required to compute the marginal probability distribution over $i$ variables. (4 points)

2 Conditional Probability (10 points)

Assume that all probabilities in $P(ABC)$ are nonzero, and suppose $P(A|BC) = P(A|C)$. Does this necessarily imply $P(B|C) = P(B|AC)$? Justify your answer with a proof, a counterexample, or a necessary and sufficient set of conditions for this to hold.

3 Bayes Nets 1 (10 points)

Recall that, by construction, in a Bayesian network, each variable is conditionally independent of its non-descendents given its parents. Provide an inductive proof that this implies:

$$P(x_1 \ldots x_n) = \Pi_{i=1}^n P(x_i|\text{parents}(x_i))$$

4 Bayes Nets 2 (20 points)

To safeguard your house, you recently installed an alarm system. There are two alarms, one in the Kitchen ($K$) and one in the Entrance ($E$). The system is modeled with the peculiar Bayesian network in Figure 1. Intuitively, Burglary ($B$) should be the source node (it should have no parents), with edges to the Kitchen and Entrance nodes. (Other edges might be appropriate too, depending upon the distribution.)

1. Based on Figure 1, provide the (conditional) probability tables of $P(B)$ (3 points), $P(K|B)$ (3 points), and $P(E|B, K)$. (3 points)
2. Transform the given Bayes net into a more natural Bayes net (using the as the Burglary as source node) based on the results of Q1. Your answer you should show the DAG and any CPTs that are required. (Hints: if you end up conditioning on an event with probability 0, that is, a specific combination of variables’ values to the right of the | has probability 0, then mark this probability as *) (5 points)

3. Based only on the network structure (i.e., not the CPTs) in Figure 1 can you guarantee that K and E independent conditional on B? (3 points) What about for your Bayes net from Q2? (3 points)