Relational Database Design Theory

Introduction to Databases

CompSci 316 Fall 2020
Announcements (Tue. Sept 22)

• HW4: due tomorrow (Wed)

• Midterm next Tuesday 09/29
  • See gradescope for policy
Today’s plan

• Start database design theory
  • Functional dependency, BCNF

• Review some concepts in between and at the end
  • Weak entity set, ISA, multiplicity, etc. in ER diagram
  • Outer joins, different join types
  • Triggers
  • EXISTS
  • Foreign keys
Motivation

- Why is UserGroup \((uid, uname, gid)\) a bad design?
  - It has redundancy—user name is recorded multiple times, once for each group that a user belongs to
    - Leads to update, insertion, deletion anomalies

- Wouldn’t it be nice to have a systematic approach to detecting and removing redundancy in designs?
  - Dependencies, decompositions, and normal forms

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... ... ...
Functional dependencies

- A **functional dependency (FD)** has the form $X \rightarrow Y$, where $X$ and $Y$ are sets of attributes in a relation $R$
- $X \rightarrow Y$ means that whenever two tuples in $R$ agree on all the attributes in $X$, they must also agree on all attributes in $Y$
FD examples

Address (street_address, city, state, zip)

• street_address, city, state → zip
• zip → city, state
• zip, state → zip?
  • This is a trivial FD
  • Trivial FD: LHS ⊇ RHS
• zip → state, zip?
  • This is non-trivial, but not completely non-trivial
  • Completely non-trivial FD: LHS ∩ RHS = ∅
Redefining “keys” using FD’s

A set of attributes $K$ is a key for a relation $R$ if

- $K \rightarrow$ all (other) attributes of $R$
  - That is, $K$ is a “super key”

- No proper subset of $K$ satisfies the above condition
  - That is, $K$ is minimal
Reasoning with FD’s

Given a relation $R$ and a set of FD’s $\mathcal{F}$

• **Does another FD follow from $\mathcal{F}$?**
  • Are some of the FD’s in $\mathcal{F}$ redundant (i.e., they follow from the others)?

• **Is $K$ a key of $R$?**
  • What are all the keys of $R$?
Attribute closure

• Given $R$, a set of FD’s $\mathcal{F}$ that hold in $R$, and a set of attributes $Z$ in $R$:
  The closure of $Z$ (denoted $Z^+$) with respect to $\mathcal{F}$ is the set of all attributes \{${A_1, A_2, ...}$\} functionally determined by $Z$ (that is, $Z \rightarrow A_1A_2 ...$)

• Algorithm for computing the closure
  • Start with closure = $Z$
  • If $X \rightarrow Y$ is in $\mathcal{F}$ and $X$ is already in the closure, then also add $Y$ to the closure
  • Repeat until no new attributes can be added
A more complex example

UserJoinsGroup (uid, uname, twitterid, gid, fromDate)

Assume that there is a 1-1 correspondence between our users and Twitter accounts

• uid → uname, twitterid
• twitterid → uid
• uid, gid → fromDate

Not a good design, and we will see why shortly
Example of computing closure

- \{gid, twitterid\}^+ = ?
- twitterid → uid
  - Add uid
  - Closure grows to \{gid, twitterid, uid\}
- uid → uname, twitterid
  - Add uname, twitterid
  - Closure grows to \{gid, twitterid, uid, uname\}
- uid, gid → fromDate
  - Add fromDate
  - Closure is now all attributes in UserJoinsGroup
Using attribute closure

Given a relation $R$ and set of FD’s $\mathcal{F}$

• Does another FD $X \rightarrow Y$ follow from $\mathcal{F}$?
  • Compute $X^+$ with respect to $\mathcal{F}$
  • If $Y \subseteq X^+$, then $X \rightarrow Y$ follows from $\mathcal{F}$

• Is $K$ a key of $R$?
  • Compute $K^+$ with respect to $\mathcal{F}$
  • If $K^+$ contains all the attributes of $R$, $K$ is a super key
  • Still need to verify that $K$ is minimal (how?)
Rules of FD’s

• Armstrong’s axioms
  • Reflexivity: If $Y \subseteq X$, then $X \rightarrow Y$
  • Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any $Z$
  • Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

• Rules derived from axioms
  • Splitting: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
  • Combining: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

Using these rules, you can prove or disprove an FD given a set of FDs

We already used these intuitive rules but check yourself again!

End of lecture Thursday 02/13
Announcements (Thu. Feb. 20)

• **Project Milestone 1:**
  - Due on Monday February 24 night
  - One report per group to be submitted to gradescope.

• More in-class labs and quizzes from next week!

• Survey to be sent soon.

• In-class quiz on Tuesday 02/25 on BCNF (to be covered today)
(Problems with) Non-key FD’s

• Consider a non-trivial FD $X \rightarrow Y$ where $X$ is *not* a super key
  • Since $X$ is not a super key, there are some attributes (say $Z$) that are not functionally determined by $X$

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y$</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$b$</td>
<td>$c_1$</td>
</tr>
<tr>
<td>$a$</td>
<td>$b$</td>
<td>$c_2$</td>
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That $b$ is associated with $a$ is recorded multiple times: redundancy, update/insertion/deletion anomaly
Example of redundancy

UserJoinsGroup \((uid, \text{uname}, \text{twitterid}, \text{gid}, \text{fromDate})\)

- \(uid \rightarrow \text{uname}, \text{twitterid}\)

(... plus other FD’s)

<table>
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<tr>
<th>(uid)</th>
<th>(\text{uname})</th>
<th>(\text{twitterid})</th>
<th>(\text{gid})</th>
<th>(\text{fromDate})</th>
</tr>
</thead>
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<tr>
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<td>Bart</td>
<td>@BartJSimson</td>
<td>dps</td>
<td>1987-04-19</td>
</tr>
<tr>
<td>123</td>
<td>Milhouse</td>
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<td>1989-12-17</td>
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<tr>
<td>456</td>
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<td>@ralphwiggum</td>
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<td>1992-09-01</td>
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<td>...</td>
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What are the problems? How do we fix them?
Decomposition

- Eliminates redundancy
- To get back to the original relation: ✗
Unnecessary decomposition

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- Fine: join returns the original relation
- Unnecessary: no redundancy is removed; schema is more complicated (and uid is stored twice!)
Bad decomposition

- Association between $gid$ and $fromDate$ is lost
- Join returns more rows than the original relation
Lossless join decomposition

- Decompose relation $R$ into relations $S$ and $T$
  - $\text{attrs}(R) = \text{attrs}(S) \cup \text{attrs}(T)$
  - $S = \pi_{\text{attrs}(S)}(R)$
  - $T = \pi_{\text{attrs}(T)}(R)$
- The decomposition is a lossless join decomposition if, given known constraints such as FD’s, we can guarantee that $R = S \bowtie T$

- Any decomposition gives $R \subseteq S \bowtie T$ (why?)
  - A lossy decomposition is one with $R \subset S \bowtie T$
Loss? But I got more rows!

- “Loss” refers not to the loss of tuples, but to the loss of information
  - Or, the ability to distinguish different original relations

No way to tell which is the original relation
Questions about decomposition

• When to decompose

• How to come up with a correct decomposition (i.e., lossless join decomposition)
An answer: BCNF

• A relation $R$ is in Boyce-Codd Normal Form if
  • For every non-trivial FD $X \rightarrow Y$ in $R$, $X$ is a super key
  • That is, all FDs follow from “key $\rightarrow$ other attributes”

• When to decompose
  • As long as some relation is not in BCNF

• How to come up with a correct decomposition
  • Always decompose on a BCNF violation (details next)
  $\therefore$ Then it is guaranteed to be a lossless join decomposition!
BCNF decomposition algorithm

• Find a BCNF violation
  • That is, a non-trivial FD $X \rightarrow Y$ in $R$ where $X$ is not a super key of $R$

• Decompose $R$ into $R_1$ and $R_2$, where
  • $R_1$ has attributes $X \cup Y$
  • $R_2$ has attributes $X \cup Z$, where $Z$ contains all attributes of $R$ that are in neither $X$ nor $Y$

• Repeat until all relations are in BCNF
BCNF decomposition example

UserJoinGroup (uid, uname, twitterid, gid, fromDate)

BCNF violation: uid → uname, twitterid

User (uid, uname, twitterid)

uid → uname, twitterid, twitterid → uid

Member (uid, gid, fromDate)

uid, gid → fromDate

BCNF
Another example

UserJoinsGroup (uid, uname, twitterid, gid, fromDate)

BCNF violation: twitterid → uid

UserId (twitterid, uid)

BCNF

UserJoinsGroup’ (twitterid, uname, gid, fromDate)

BCNF violation: twitterid → uname

twitterid → uname

twitterid, gid → fromDate

UserName (twitterid, uname)

BCNF

Member (twitterid, gid, fromDate)

BCNF
Why is BCNF decomposition lossless

Given non-trivial $X \rightarrow Y$ in $R$ where $X$ is not a super key of $R$, need to prove:

- Anything we project always comes back in the join:
  \[ R \subseteq \pi_{XY}(R) \bowtie \pi_{XZ}(R) \]
  - Sure; and it doesn’t depend on the FD

- Check and prove yourself!

- Anything that comes back in the join must be in the original relation:
  \[ R \supseteq \pi_{XY}(R) \bowtie \pi_{XZ}(R) \]
  - Proof will make use of the fact that $X \rightarrow Y$
Recap

• Functional dependencies: a generalization of the key concept
• Non-key functional dependencies: a source of redundancy
• BCNF decomposition: a method for removing redundancies
  • BNCF decomposition is a lossless join decomposition
• BCNF: schema in this normal form has no redundancy due to FD’s
Summary

• Philosophy behind BCNF: 
  Data should depend on the key, 
  the whole key, 
  and nothing but the key!
  • You could have multiple keys though

• Other normal forms
  • 4NF and Multi-valued-dependencies: later in the course
  • Not covered
    • 3NF: More relaxed than BCNF; will not remove redundancy if doing so makes FDs harder to enforce
    • 2NF: Slightly more relaxed than 3NF
    • 1NF: All column values must be atomic