Getting in front

- Suppose we want to add a new element
  - At the back of a string or an ArrayList or a ...
  - At the front of a string or an ArrayList or a ...
  - Is there a difference? Why? What's complexity?

- Suppose this is an important problem: we want to grow at the front (and perhaps at the back)
  - Think editing film clips and film splicing
  - Think DNA and gene splicing

- Self-referential data structures to the rescue
  - References, reference problems, recursion

ArrayLists and linked lists as ADTs

- As an ADT (abstract data type) ArrayLists support
  - Constant-time or O(1) access to the k-th element
  - Amortized linear or O(n) storage/time with add
    - Total storage used in n-element vector is approx. 2n, spread over all accesses/additions (why?)
    - Adding a new value in the middle of an ArrayList is expensive, linear or O(n) because shifting required

- Linked lists as ADT
  - Constant-time or O(1) insertion/deletion anywhere, but...
  - Linear or O(n) time to find where, sequential search

- Good for sparse structures: when data are scarce, allocate exactly as many list elements as needed, no wasted space/copying (e.g., what happens when vector grows?)

Linked list applications

- Remove element from middle of a collection, maintain order, no shifting. Add an element in the middle, no shifting
  - What's the problem with a vector (array)?
  - Naively keep characters in a linked list, but in practice too much storage, need more esoteric data structures

- What's $(3x^5 + 2x^3 + x + 5) + (2x^4 + 5x^3 + x^2 + 4x)$ ?
  - As a vector $(3, 0, 2, 0, 1, 5)$ and $(0, 2, 5, 1, 4, 0)$
  - As a list $((3, 5), (2, 3), (1, 1), (5, 0))$ and ________?
  - Most polynomial operations sequentially visit terms, don't need random access, do need "splicing"

- What about $(3x^{100} + 5)$ ?

Linked list applications continued

- If programming in C, there are no "growable-arrays", so typically linked lists used when # elements in a collection varies, isn't known, can't be fixed at compile time
  - Could grow array, potentially expensive/wasteful especially if # elements is small.
  - Also need # elements in array, requires extra parameter
  - With linked list, one pointer used to access all the elements in a collection

- Simulation/modelling of DNA gene-splicing
  - Given list of millions of CGTA... for DNA strand, find locations where new DNA/gene can be spliced in
    - Remove target sequence, insert new sequence
Linked lists, CDT and ADT

- As an ADT
  - A list is empty, or contains an element and a list
  - ( ) or (x, (y, ()))

- As a picture

  ![Diagram of a linked list](image)

- As a CDT (concrete data type)
  ```java
  public class Node {
      String info;
      Node next;
      Node(String s, Node link) {
          info = s;
          next = link;
      }
  }
  ```

Building linked lists

- Add words to the front of a list (draw a picture)
  - Create new node with next pointing to list, reset start of list
  ```java
  public class Node {
      String info;
      Node next;
      Node(String s, Node link) {
          info = s;
          next = link;
      }
  }
  ```
  ```java
  Node list = null;
  while (scanner.hasNext()) {
      list = new Node(scanner.nextString(), list);
  }
  ```

  What about adding to the end of the list?

Dissection of add-to-front

- List initially empty
- First node has first word
- Each new word causes new node to be created
  - New node added to front
- Rhs of operator = completely evaluated before assignment

  ![Diagram of add-to-front](image)

Standard list processing (iterative)

- Visit all nodes once, e.g., count them or process them
  ```java
  public int size(Node list) {
      int count = 0;
      while (list != null) {
          count++;
          list = list.next;
      }
      return count;
  }
  ```

  What changes in code above if we change what “process” means?
  - Print nodes?
  - Append “s” to all strings in list?
Nancy Leveson: Software Safety

Founded the field

- Mathematical and engineering aspects
  - Air traffic control
  - Microsoft word

"C++ is not state-of-the-art, it’s only state-of-the-practice, which in recent years has been going backwards"

- Software and steam engines: once extremely dangerous?
- THERAC 25: Radiation machine that killed many people

See Splice.java

- Consider prepending (add to front) and two methods:
  ```java
  public void prepend(String s) {
      myString = s + myString;
  }
  public void prepend(String s) {
      myFront = new Node(s, myFront);
      myCount += s.length();
  }
  ```

- What is hidden complexity of these operations? Why?

Timings in Splice.java

<table>
<thead>
<tr>
<th>method length</th>
<th>StringStrand</th>
<th>LinkStrand</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>4.253</td>
<td>0.001</td>
</tr>
<tr>
<td>108,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>7.028</td>
<td>0.001</td>
</tr>
<tr>
<td>135,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td>11.133</td>
<td>0.001</td>
</tr>
<tr>
<td>162,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7000</td>
<td>16.418</td>
<td>0.001</td>
</tr>
<tr>
<td>189,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8000</td>
<td>??</td>
<td>??</td>
</tr>
<tr>
<td>216,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

New task in Strand.java

- Rather than simply prepending, what about splicing anywhere?
  - We have `s.insert(k, str)` to add string at $k^{th}$ position, so prepending is `s.insert(0, str)`

- We want to mirror this behavior in all classes
  - What do we do in base class?
  - How do we implement in LinkStrand class?
    - What are issues?
    - How fast will it be?
Building linked lists continued

- What about adding a node to the end of the list?
  - Can we search and find the end?
  - If we do this every time, what’s complexity of building an N-node list? Why?

- Alternatively, keep pointers to first and last nodes of list
  - If we add node to end, which pointer changes?
  - What about initially empty list: values of pointers?
    - Will lead to consideration of header node to avoid special cases in writing code

- What about keeping list in order, adding nodes by splicing into list? Issues in writing code? When do we stop searching?

Standard list processing (recursive)

- Visit all nodes once, e.g., count them

  ```java
  public int recsize(Node list) {
    if (list == null) return 0;
    return 1 + recsize(list.next);
  }
  ```

- Base case is almost always empty list: null pointer
  - Must return correct value, perform correct action
  - Recursive calls use this value/state to anchor recursion
  - Sometimes one node list also used, two “base” cases

- Recursive calls make progress towards base case
  - Almost always using list.next as argument

Recursion and linked lists

- Print nodes in reverse order
  - Print all but first node and...
    - Print first node before or after other printing?

  ```java
  public void print(Node list) {
    if (list != null) {
      System.out.println(list.info);
      print(list.next);
    }
  }
  ```
Complexity Practice

✧ What is complexity of Build? (what does it do?)

```java
public Node build(int n) {
    if (null == n) return null;
    Node first = new Node(n, build(n-1));
    for(int k = 0; k < n-1; k++) {
        first = new Node(n, first);
    }
    return first;
}
```

✧ What does this code do? How can we reason about it?
- Empty list, one-node list, two-node list, n-node list
- Similar to proof by induction

Write an expression for $T(n)$ and for $T(0)$, solve.
- Let $T(n)$ be time for build to execute with n-node list
- $T(n) = T(n-1) + O(n)$

Changing a linked list recursively

✧ Pass list to method, return altered list, assign to list
- Idiom for changing value parameters

```java
list = change(list, "apple");
public Node change(Node list, String key) {
    if (list != null) {
        list.next = change(list.next, key);
        if (list.info.equals(key)) return list.next;
    } else return list;
    return null;
}
```

- What does this code do? How can we reason about it?
  - Empty list, one-node list, two-node list, n-node list