

First Homework Assignment

Write the solution to each question on a single page. The deadline for handing in solutions is January 26.

Question 1. (20 = 10 + 10 points). If n basketball teams play each other team exactly once, how many games will be played in total? If the teams then compete in a single elimination tournament (similar to March Madness), how many additional games are played?

Question 2. (20 = 10 + 10 points).

- (a) (Problem 1.2-7 in our textbook). Let $|D| = |R| = n$. Show that the following statement is true: The function $f : D \rightarrow R$ is surjective if and only if f is injective.
- (b) Is the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3x + 2$ a bijection? Prove or give a counterexample.

Question 3. (20 = 6 + 7 + 7 points).

- (a) What is the coefficient of the x^8 term of $(x - 2)^{30}$?
- (b) What is the coefficient of the $x^i y^j z^k$ term of $(x + y + z)^n$?
- (c) Show that $\binom{n}{k} = \binom{n}{n-k}$.

Question 4. (20 = 6+7+7 points). For (a) and (b), prove or disprove that the relations given are equivalence relations. For (c), be sure to justify your answer.

- (a) Choose some $k \in \mathbb{Z}$. Let $x, y \in \mathbb{Z}$. We say $x \sim y$ if $x \equiv y \pmod{k}$.
- (b) Let x, y be positive integers. We say $x \sim y$ if the greatest common factor of x and y is greater than 1.
- (c) How many ways can you distribute k identical cookies to n children?