

## Fifth Homework Assignment

Write the solution to each problem on a single page. The deadline for handing in solutions is 8 April 2009.

**Question 1.** (20 points). Use the Principle of Inclusion-Exclusion to count the surjective functions  $f : M \rightarrow N$ , where both sets are finite with  $m = |M|$  and  $n = |N|$ .

**Question 2.** (20 = 6 + 7 + 7 points). (Problems 5.3-1 to 3 in our textbook). Suppose you have a fair coin, one in which a flip gives head with probability one half and tail with probability one half. You do three flips with this coin.

- What is the probability that two flips in a row are heads, given that there is an even number of heads?
- Is the event that two flips in a row are heads independent of the event that there is an even number of heads?
- Is the event of getting at most one tail independent of the event that not all flips are identical?

**Question 3.** (20 points). (Problem 5.4-16 in our textbook). Suppose you have two nickels, two dimes, and two quarters in a bag. You draw three coins from the bag, without replacement. What is the expected amount of money you get?

**Question 4.** (20 = 6 + 7 + 7 points). (Problem 5.5-8 in our textbook). Suppose you hash  $n$  items into  $k$  locations.

- What is the probability that all  $n$  items hash to different locations?
- What is the probability that the  $i$ -th item gives the first collision?
- What is the expected number of items you hash until the first collision?

**Question 5.** (20 = 7 + 7 + 6 points). In the programming language of your choice, write the following two functions:

- GETMEAN
- GETVARIANCE

These methods should take an array of values as input (the experimental results for each trial) and return a floating point number. Then, flip a coin 20 times (or simulate this on the computer) and use these methods to compute the mean and the variance of your trials. Are the results what you would have expected?

**Question 6.** (20 = 10 + 10 points). (Problems 5.7-8 and 14 in our textbook).

- Show that if  $X$  and  $Y$  are independent, and  $b$  and  $c$  are constant, then  $X - b$  and  $Y - c$  are independent.
- Given a random variable  $X$ , how does the variance of  $cX$  relate to that of  $X$ ?