CPS 170: Artificial Intelligence

http://www.cs.duke.edu/courses/spring09/cps170/

Game Theory

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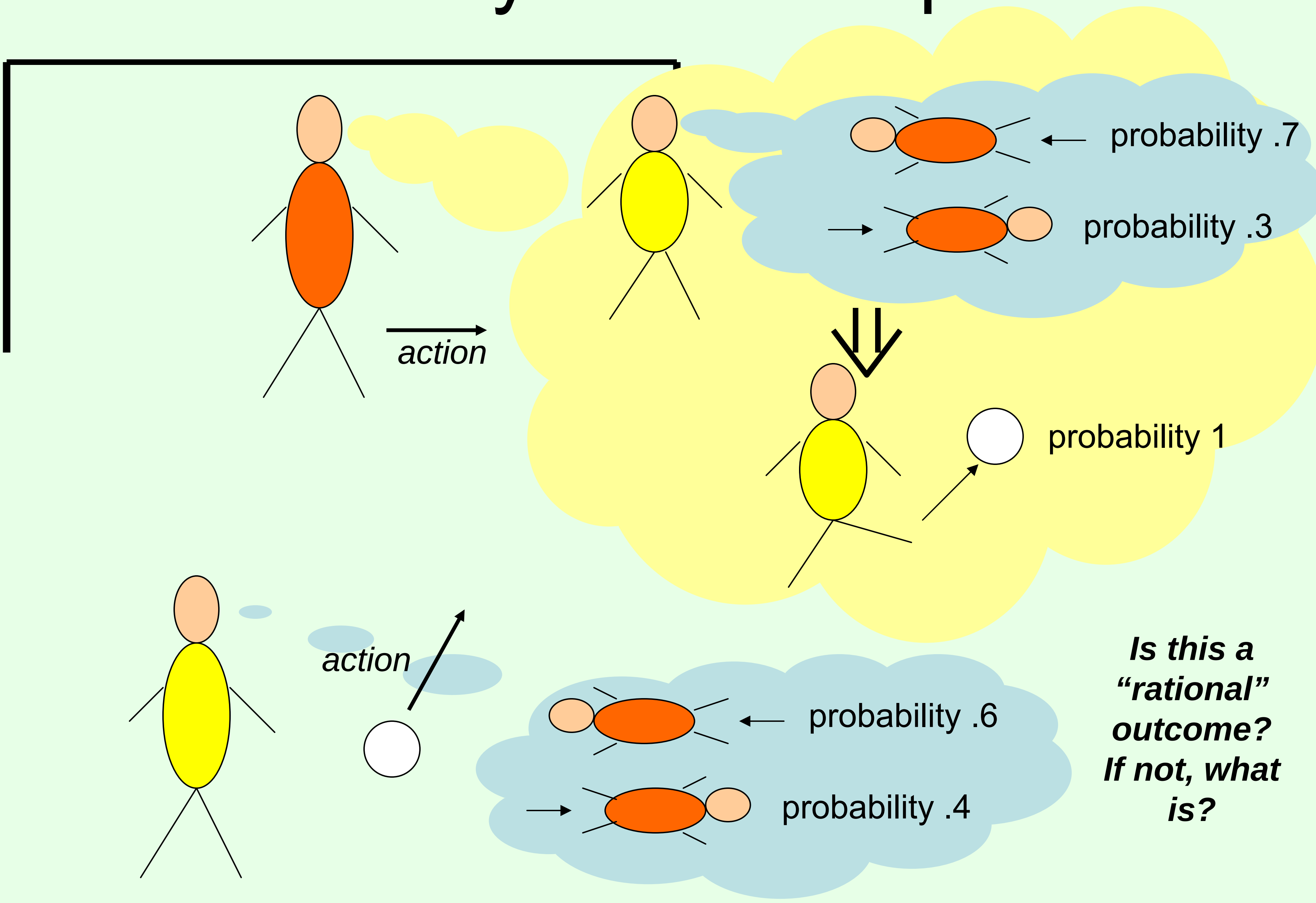
What is game theory?

- Game theory studies settings where multiple parties (agents) each have
  - different preferences (utility functions),
  - different actions that they can take
- Each agent's utility (potentially) depends on all agents' actions
  - What is optimal for one agent depends on what other agents do
    - Very circular!
- Game theory studies how agents can rationally form beliefs over what other agents will do, and (hence) how agents should act
  - Useful for acting as well as predicting behavior of others

Penalty kick example

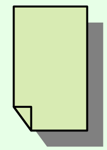
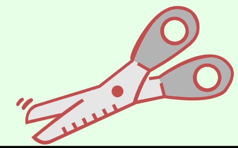
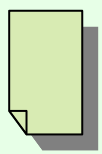
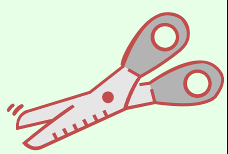


Is this a "rational" outcome? If not, what is?

Rock-paper-scissors

Column player aka. player 2 (simultaneously) chooses a column

Row player aka. player 1 chooses a row

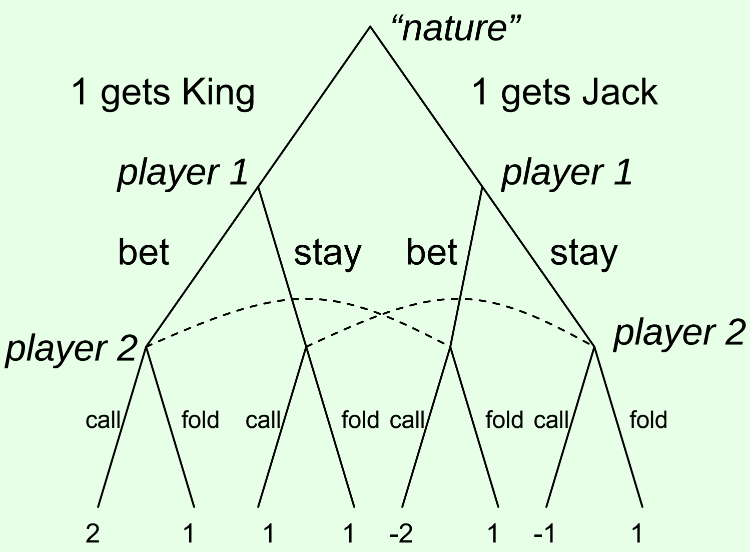
			
	0, 0	-1, 1	1, -1
	1, -1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

A row or column is called an action or (pure) strategy

Row player's utility is always listed first, column player's second

Zero-sum game: the utilities in each entry sum to 0 (or a constant)
Three-player game would be a 3D table with 3 utilities per entry, etc.

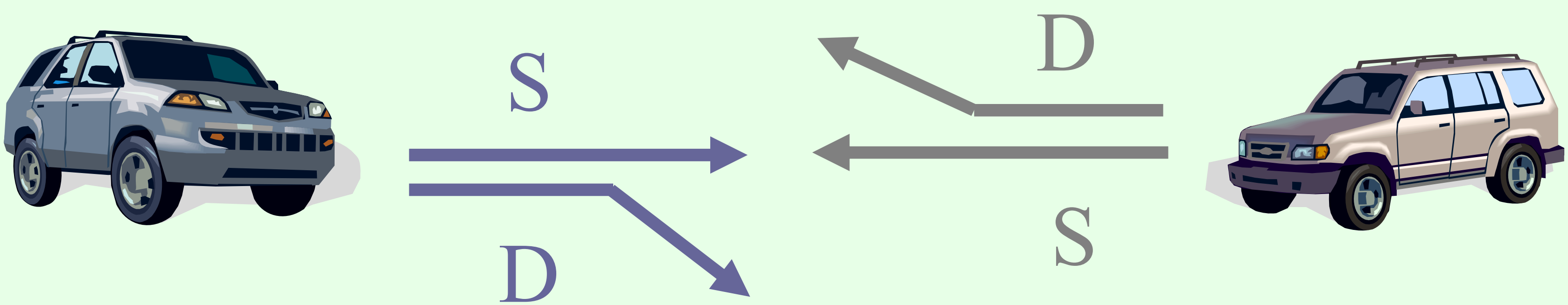
A poker-like game



	cc	cf	fc	ff
bb	0, 0	0, 0	1, -1	1, -1
bs	.5, -.5	1.5, -1.5	0, 0	1, -1
sb	-.5, .5	-.5, .5	1, -1	1, -1
ss	0, 0	1, -1	0, 0	1, -1

“Chicken”

- Two players drive cars towards each other
- If one player goes straight, that player wins
- If both go straight, they both die

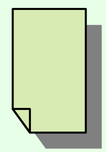
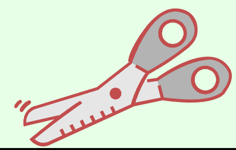
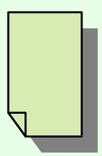
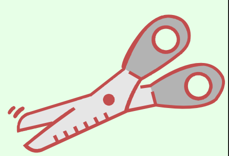


	D	S
D	0, 0	-1, 1
S	1, -1	-5, -5

not zero-sum

Rock-paper-scissors – Seinfeld variant

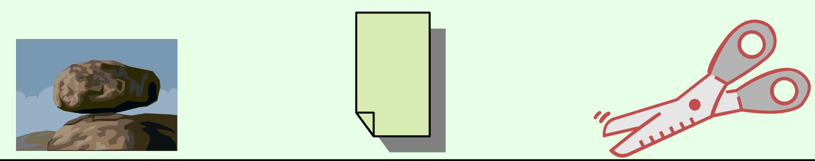
MICKEY: All right, rock beats paper!
(Mickey smacks Kramer's hand for losing)
KRAMER: I thought paper covered rock.
MICKEY: Nah, rock flies right through paper.
KRAMER: What beats rock?
MICKEY: (looks at hand) Nothing beats rock.

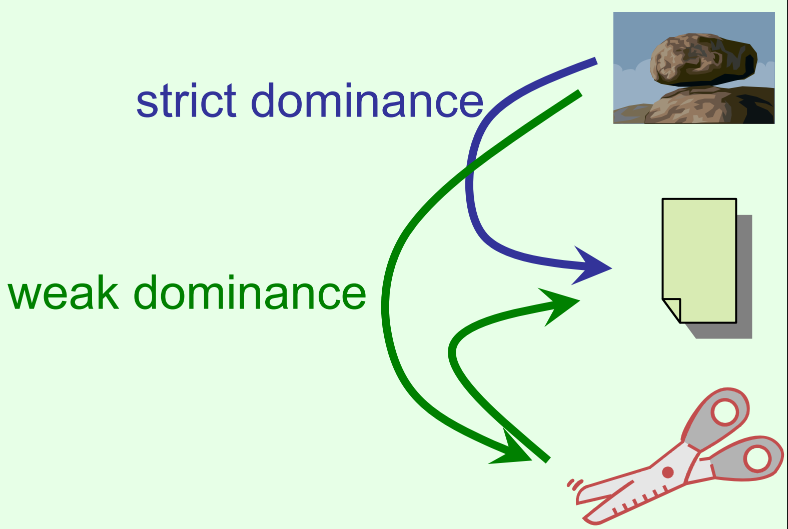
			
	0, 0	1, -1	1, -1
	-1, 1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

Dominance

- Player i's strategy s_i **strictly dominates** s_i' if
  - for any s_{-i}, $u_i(s_i , s_{-i}) > u_i(s_i', s_{-i})$
- s_i **weakly dominates** s_i' if
  - for any s_{-i}, $u_i(s_i , s_{-i}) \geq u_i(s_i', s_{-i})$; and
  - for some s_{-i}, $u_i(s_i , s_{-i}) > u_i(s_i', s_{-i})$

-i = "the player(s) other than i"





	0, 0	1, -1	1, -1
	-1, 1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

Prisoner's Dilemma

- Pair of criminals has been caught
- District attorney has evidence to convict them of a minor crime (1 year in jail); knows that they committed a major crime together (3 years in jail) but cannot prove it
- Offers them a deal:
  - If both confess to the major crime, they each get a 1 year reduction
  - If only one confesses, that one gets 3 years reduction

	confess	don't confess
confess	-2, -2	0, -3
don't confess	-3, 0	-1, -1

“Should I buy an SUV?”

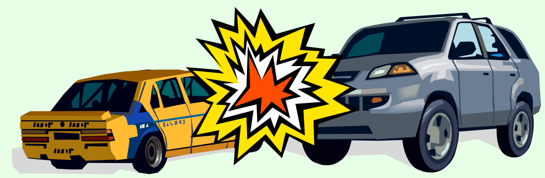
purchasing + gas cost


cost: 5


cost: 3

accident cost

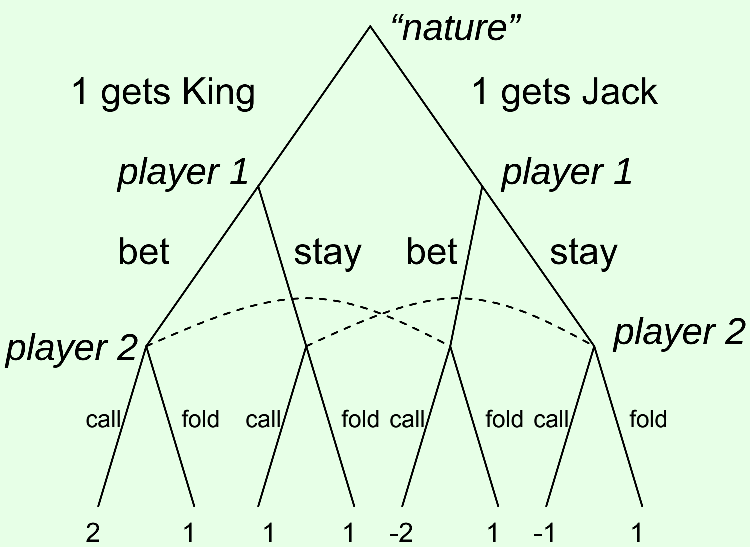
cost: 5  cost: 5

cost: 8  cost: 2

cost: 5  cost: 5

		
	-10, -10	-7, -11
	-11, -7	-8, -8

A poker-like game



	cc	cf	fc	ff
bb	0, 0	0, 0	1, -1	1, -1
bs	.5, -.5	1.5, -1.5	0, 0	1, -1
sb	-.5, .5	-.5, .5	1, -1	1, -1
ss	0, 0	1, -1	0, 0	1, -1

“2/3 of the average” game

- Everyone writes down a number between 0 and 100
- Person closest to 2/3 of the average wins
- Example:
  - A says 50
  - B says 10
  - C says 90
  - Average(50, 10, 90) = 50
  - 2/3 of average = 33.33
  - A is closest (|50-33.33| = 16.67), so A wins

Iterated dominance

- Iterated dominance: remove (strictly/weakly) dominated strategy, repeat
- Iterated strict dominance on Seinfeld's RPS:

	0, 0	1, -1	1, -1
	-1, 1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

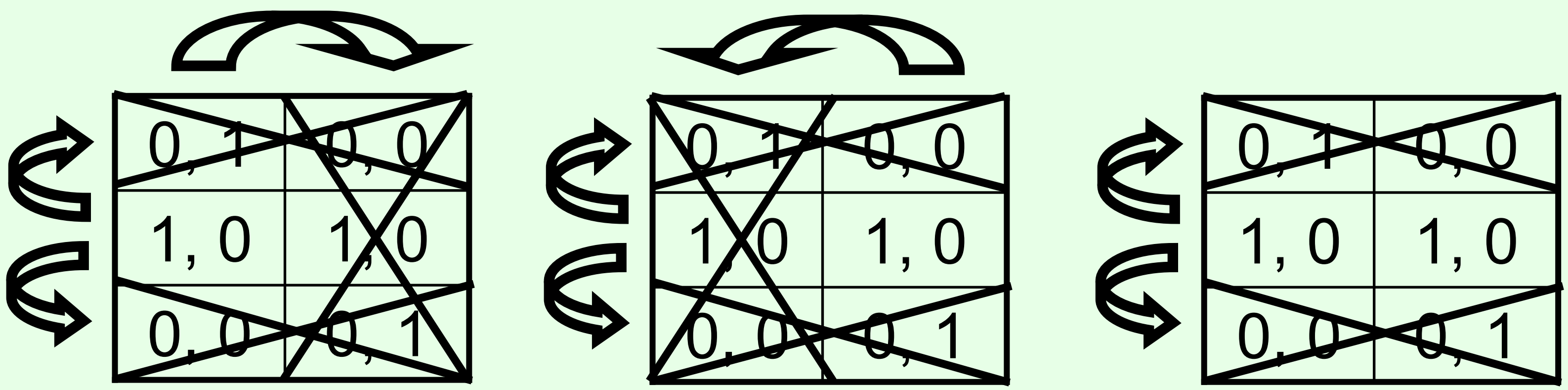
→

	0, 0	1, -1
	-1, 1	0, 0

Iterated dominance: path (in)dependence

Iterated weak dominance is path-dependent: sequence of eliminations may determine which solution we get (if any)

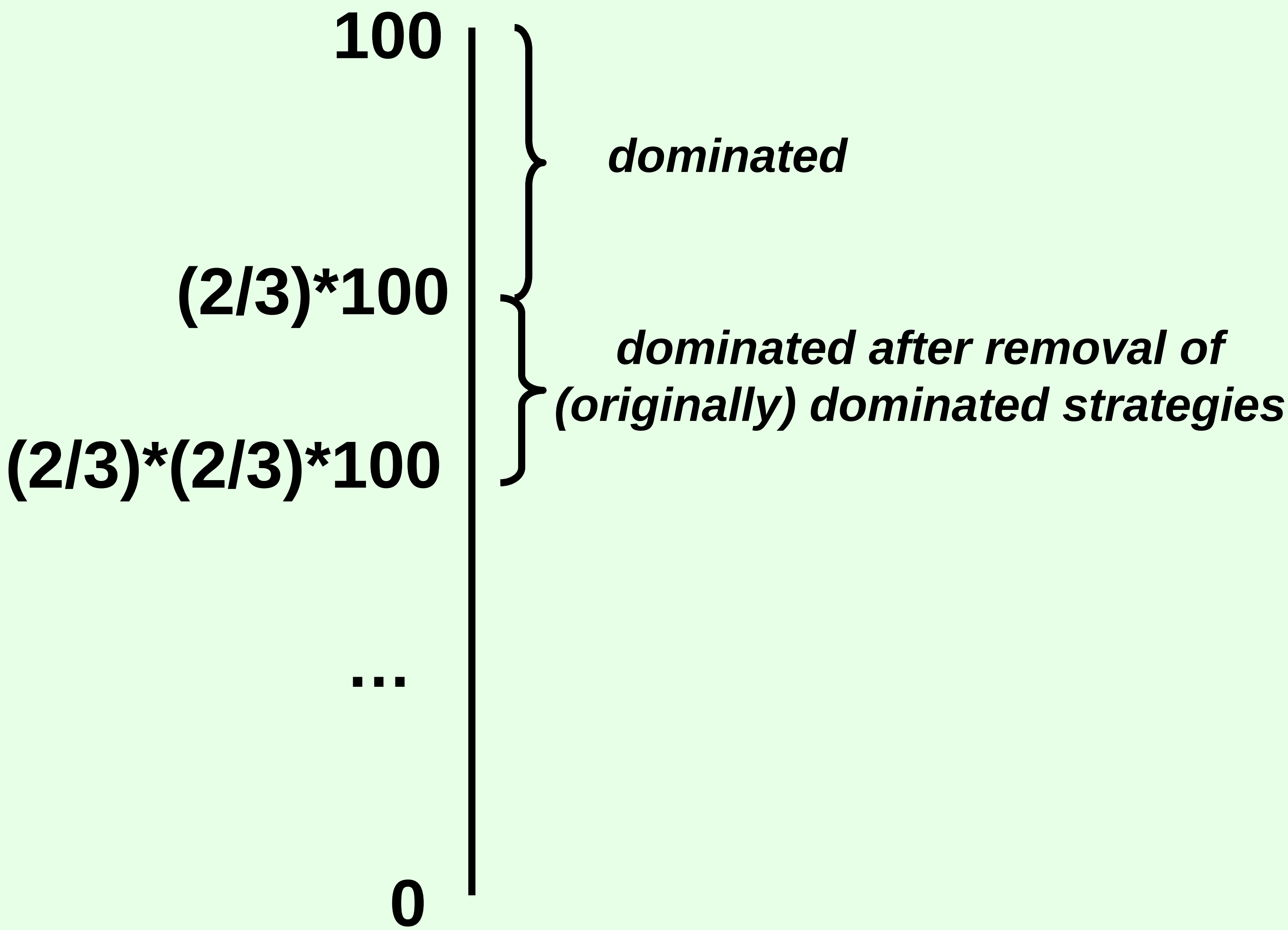
(whether or not dominance by mixed strategies allowed)



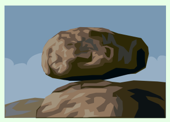
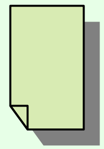
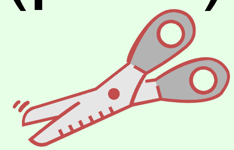
Iterated strict dominance is path-independent: elimination process will always terminate at the same point

(whether or not dominance by mixed strategies allowed)

“2/3 of the average” game revisited



Mixed strategies

- Mixed strategy for player i = probability distribution over player i's (pure) strategies
- E.g. 1/3  , 1/3  , 1/3 
- Example of dominance by a mixed strategy:

1/2	3, 0	0, 0
1/2	0, 0	3, 0
	1, 0	1, 0

Checking for dominance by mixed strategies

- Linear program for checking whether strategy s_i^* is strictly dominated by a mixed strategy:
- maximize ε
- such that:
  - for any s_{-i}, $\Sigma_{s_i} \mathbf{p}_{s_i} u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i}) + \varepsilon$
  - $\Sigma_{s_i} \mathbf{p}_{s_i} = 1$

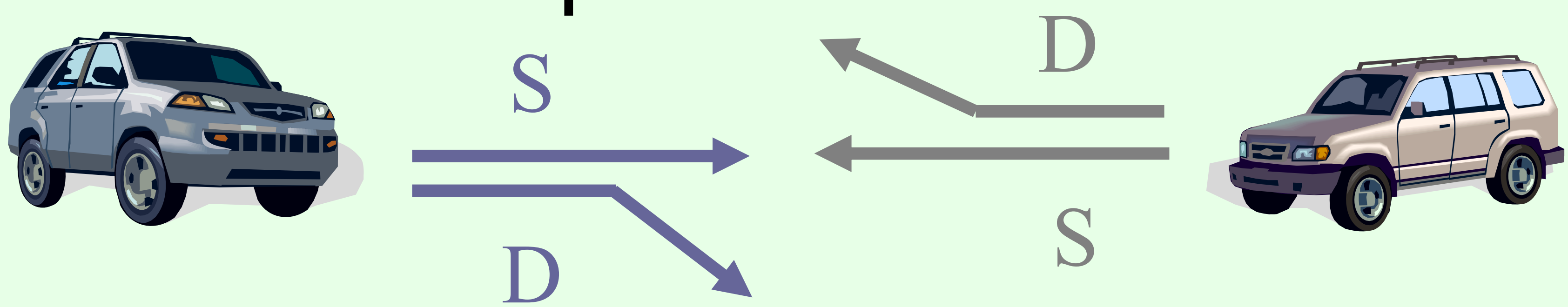
- Linear program for checking whether strategy s_i^* is weakly dominated by a mixed strategy:
- maximize $\Sigma_{s_{-i}}(\Sigma_{s_i} \mathbf{p}_{s_i} u_i(s_i, s_{-i})) - u_i(s_i^*, s_{-i})$
- such that:
  - for any s_{-i}, $\Sigma_{s_i} \mathbf{p}_{s_i} u_i(s_i, s_{-i}) \geq u_i(s_i^*, s_{-i})$
  - $\Sigma_{s_i} \mathbf{p}_{s_i} = 1$

Nash equilibrium [Nash 50]

- A vector of strategies (one for each player) is called a strategy profile
- A strategy profile $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ is a Nash equilibrium if each σ_i is a best response to σ_{-i}
  - That is, for any i, for any σ_i', $u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i})$
- Note that this does not say anything about multiple agents changing their strategies at the same time
- In any (finite) game, at least one Nash equilibrium (possibly using mixed strategies) exists [Nash 50]

- (Note - singular: equilibrium, plural: equilibria)

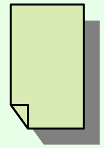
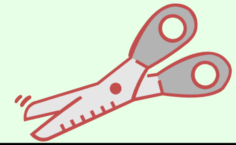
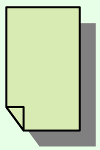
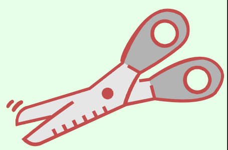
Nash equilibria of “chicken”



	D	S
D	0, 0	-1, 1
S	1, -1	-5, -5

- (D, S) and (S, D) are Nash equilibria
  - They are pure-strategy Nash equilibria: nobody randomizes
  - They are also strict Nash equilibria: changing your strategy will make you strictly worse off
- No other pure-strategy Nash equilibria

Rock-paper-scissors

			
	0, 0	-1, 1	1, -1
	1, -1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

- Any pure-strategy Nash equilibria?
- But it has a mixed-strategy Nash equilibrium:

  Both players put probability 1/3 on each action
- If the other player does this, every action will give you expected utility 0
  - Might as well randomize

Nash equilibria of “chicken”…

	D	S
D	0, 0	-1, 1
S	1, -1	-5, -5

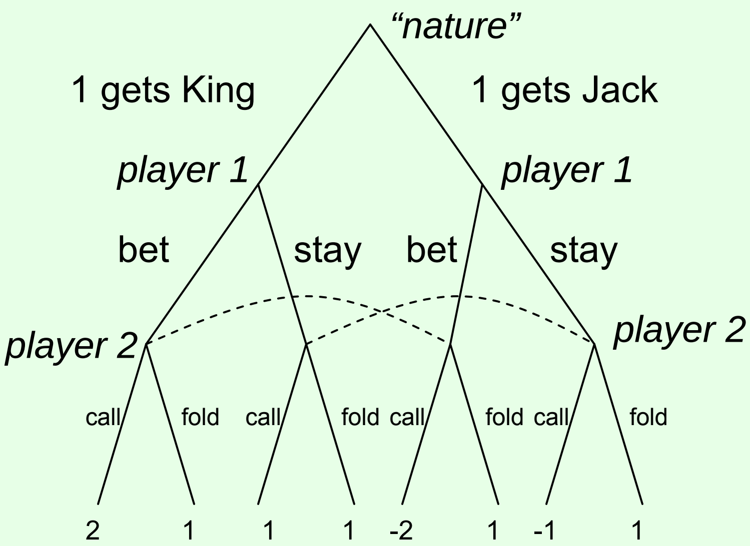
- Is there a Nash equilibrium that uses mixed strategies? Say, where player 1 uses a mixed strategy?
- If a mixed strategy is a best response, then all of the pure strategies that it randomizes over must also be best responses
- So we need to make player 1 indifferent between D and S
- Player 1’s utility for playing D = $-p^c_S$
- Player 1’s utility for playing S = $p^c_D - 5p^c_S = 1 - 6p^c_S$
- So we need $-p^c_S = 1 - 6p^c_S$ which means $p^c_S = 1/5$
- Then, player 2 needs to be indifferent as well
- Mixed-strategy Nash equilibrium: ((4/5 D, 1/5 S), (4/5 D, 1/5 S))
  - People may die! Expected utility -1/5 for each player

The presentation game

		Presenter: *Put effort into presentation (E)*	Presenter: *Do not put effort into presentation (NE)*
Audience	*Pay attention (A)*	4, 4	-16, -14
	Do not pay attention (NA)	0, -2	0, 0

- Pure-strategy Nash equilibria: (A, E), (NA, NE)
- Mixed-strategy Nash equilibrium:
  ((1/10 A, 9/10 NA), (4/5 E, 1/5 NE))
  - Utility 0 for audience, -14/10 for presenter
  - Can see that some equilibria are strictly better for both players than other equilibria, i.e. some equilibria Pareto-dominate other equilibria

A poker-like game



		2/3 cc	cf	1/3 fc	ff
1/3	bb	0, 0	0, 0	1, -1	1, -1
2/3	bs	.5, -.5	1.5, -1.5	0, 0	1, -1
	sb	-.5, .5	-.5, .5	1, -1	1, -1
	ss	0, 0	1, -1	0, 0	1, -1

- To make player 1 indifferent between bb and bs, we need:
  utility for bb = 0*P(cc)+1*(1-P(cc)) = .5*P(cc)+0*(1-P(cc)) = utility for bs
  That is, P(cc) = 2/3
- To make player 2 indifferent between cc and fc, we need:
  utility for cc = 0*P(bb)+(-.5)*(1-P(bb)) = -1*P(bb)+0*(1-P(bb)) = utility for fc
  That is, P(bb) = 1/3