CPS 590.4 - Computational Microeconomics: Game Theory, Social Choice, and Mechanism Design

Homework 1 (due Feb. 12)

Note the rules for assignments on the course web page. Show all your work, but circle your final answer. Contact Andrew (kephart@cs.duke.edu) or Vince (conitzer@cs.duke.edu) with any questions.

1. (Risk attitudes.) Bob is making plans for Spring Break. He most prefers to go to Cancun, a trip that would cost him \$3500. Another good option is to go to Miami, which would cost him only \$1000. Bob is really excited about Spring Break and cares about nothing else in the world right now. As a result, Bob's utility u as a function of his budget b is given by:

- $u(b) = 0$ for $b < \$1000$;
- $u(b) = 1$ for $\$1000 \leq b < \$3500$;
- $u(b) = 2$ for $b \geq \$3500$.

Bob's budget right now is \$1500 (which would give him a utility of 1, for going to Miami).

Bob's wealthy friend Alice is aware of Bob's predicament and wants to offer him a "fair gamble." Define a *fair gamble* to be a random variable with expected value \$0. An example fair gamble (with two outcomes) is the following: \$-150 with probability 2/5, and \$100 with probability 3/5. If Bob were to accept this gamble, he would end up with \$1350 with probability 2/5, and with \$1600 with probability 3/5. In either case, Bob's utility is still 1, so Bob's expected utility for accepting this gamble is $(2/5) \cdot (1) + (3/5) \cdot (1) = 1$.

a (5 points). Find a fair gamble with two outcomes that would strictly increase Bob's expected utility.

b (5 points). Find a fair gamble with two outcomes that would strictly decrease Bob's expected utility.

2. (Estimating utilities (30 points).)

There are four things we know about Edoardo Umberto Massimo: he is (1) Italian, (2) attractive, (3) the kind of person you want on your team, and (4) an expected utility maximizer. Naturally, we are most fascinated by (4)—which, in any case, may help to explain (2) and (3). When presented with two distributions over a set of possible outcomes, E.U.M. says, without hesitation, which he prefers, and you will not catch him in any inconsistencies.

We have four outcomes, A, B, C, D. We will accordingly represent distributions as vectors of four probabilities. $(p_A, p_B, p_C, p_D) \succ (p'_A, p'_B, p'_C, p'_D)$ will denote that E.U.M. prefers distribution p to p'. We learn the following four preferences:

- $(.1, .2, .3, .4) \succ (.1, .2, .4, .3)$
- $(.4, .4, .1, .1) \succ (.4, .2, .2, .2)$
- $(.6, .1, 0, .3) \succ (.4, .3, .3, 0)$
- $(.4, .3, .2, .1) \succ (.5, .5, 0, 0)$

Obviously, we jump on the opportunity to estimate the utilities of this fascinating man.

Our goal will be to assign utilities in the interval $[0, 1]$ to the four outcomes that are consistent with E.U.M.'s preferences. Write a linear program formulation for this. You should add an objective to satisfy the consistency constraints by as large a margin as possible (similar to our linear program for strict dominance by mixed strategies). You may either: **(option 1)** write the general version of the problem (for arbitrary outcome sets, preferences, and probabilities) in mathematical notation AND use whatever solver you like to solve the above instance, or **(option 2)** use the MathProg (.mod) language to model the general problem and solve the specific instance all at once. (Hint: the optimal objective value is 0.02.) Attach your model and the solver's output. Which one of the four constraints (corresponding to the above four preferences) has slack in the optimal solution, i.e., it is satisfied by a greater margin than needed?

3. (Normal-form games.)

a (10 points). The following game has a unique Nash equilibrium. Find it, and prove that it is unique. (Hint: look for strict dominance.)

5, 0	1, 2	4, 0
2, 4	2, 4	3, 5
0, 1	4, 0	4, 0

b (10 points). Construct a single 2×2 normal-form game that simultaneously has all four of the following properties.

1. The game is not solvable by weak dominance (at least one player does not have a weakly dominant strategy).
2. The game is solvable by iterated weak dominance (so that one pure strategy per player remains).
3. In addition to the iterated weak dominance solution (which is a Nash equilibrium), there is a second pure-strategy Nash equilibrium.

4. Both players strictly prefer the second equilibrium to the first.

(Hints: the second pure-strategy equilibrium should not be strict; the pure-strategy equilibria should be in opposite corners of the matrix.) If you cannot get all four properties, construct an example with as many of the properties as you can.

c (10 points). Consider the following game:

5, 5	3, 6
6, 3	0, 0

Find a correlated equilibrium that places positive probability on all entries of the matrix, except the lower-right hand entry. Try to maximize the probability in the upper-left hand entry.

4. (Extensive-form games.) Consider the game below.

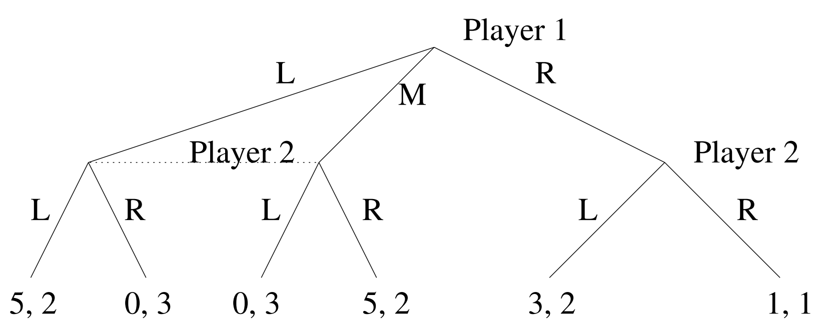

Figure 1: An extensive-form game with imperfect information.

a (10 points). Give the normal-form representation of this game.
b (10 points). Give a Nash equilibrium where player 1 sometimes plays left. (Remember that you must specify each player's strategy at *every* information set.)
c (10 points). Characterize the subgame perfect equilibria of the game. (Remember that you must specify each player's strategy at *every* information set.)