Crowdsourcing Societal Tradeoffs
Homework 2

January 23, 2015

Due: February 4, 2015

Please see the rules for homework on the course website, and contact Rupert, Markus, and/or Vince with any questions.

1. Recall that a *profile* is a vector of votes, namely one vote for each voter. I.e., it consists of all the votes cast in the election. (If the rule is *anonymous*, meaning it treats all voters the same, then it does not matter who cast which vote, so we can treat it as a *multiset* of votes, which simply specifies how often each possible ranking of the candidates is cast as a vote.)

   Specify a *single* profile for which the voting rules plurality, Borda, and Copeland all produce different winners. Please keep your profile as simple as possible.

2. In this exercise, we consider a criterion for voting rules called *reinforcement*. Informally, reinforcement means the following. Suppose profile of votes V_1 results in alternative a winning. Moreover, suppose that profile of votes V_2 results in the *same* alternative a winning. Then, when we combine all these votes to obtain a bigger profile $V_1 + V_2$, a should still win.[1]

   For example, suppose V_1 consists of three votes, $a \succ b \succ c$, $a \succ c \succ b$, and $b \succ c \succ a$. Also suppose V_2 consists of three votes, $a \succ b \succ c$, $a \succ c \succ b$, and $c \succ b \succ a$. Then, under plurality, a wins both for V_1 and for V_2. It also wins for $V_1 + V_2$, because it gets 4 points from these 6 votes whereas the other two alternatives only get 1 each. But this is not yet a proof that plurality satisfies reinforcement; for a rule to satisfy reinforcement, the condition must hold for *all* V_1 and V_2.

   (a) **Show** that plurality satisfies reinforcement.

[1] Formally, a voting rule f—which may output more than one alternative in the case of a tie—satisfies reinforcement if $f(V_1) \cap f(V_2) \neq \emptyset$ implies $f(V_1 + V_2) = f(V_1) \cap f(V_2)$.

(b) **Show** that maximin does not satisfy reinforcement. To do so it suffices to give an example of two sets V_1 and V_2 such that a wins for each of V_1 and V_2, but not for $V_1 + V_2$. Again, please keep your example as simple as possible.

3. This last exercise is based on an idea that came up in the wiki discussion on the consistency of tradeoffs. One way to guarantee consistent tradeoffs is to only look at a subset of tradeoffs, and complete the remaining tradeoffs in a consistent way. For example, if there are three activities a, b, and c, and we know the tradeoff between a and b and the tradeoff between b and c, there is only one possibility for the tradeoff between a and c that makes the collection of tradeoffs consistent. So we simply let voters report their tradeoffs for a vs. b and b vs. c only, take the median in each case, and infer the consistent value for a vs. c. Of course, we also could have voted over a vs. b and a vs. c (and inferred b vs. c), or a vs. c and b vs. c (and inferred a vs. b).[2]

   There is, however, a downside. **Show** (by giving an example) that, when voters individually have consistent tradeoffs and vote truthfully on the pairs on which they are asked to vote, the choice of which pairs to vote on can affect the outcome (say, the outcome of a vs. b).

[2] In general (for more than three alternatives), every *spanning tree* of the complete graph gives rise to one way of ensuring consistent tradeoffs, by only voting on tradeoffs corresponding to tree edges and completing remaining tradeoffs in a consistent way.