A better rule for aggregating societal tradeoffs

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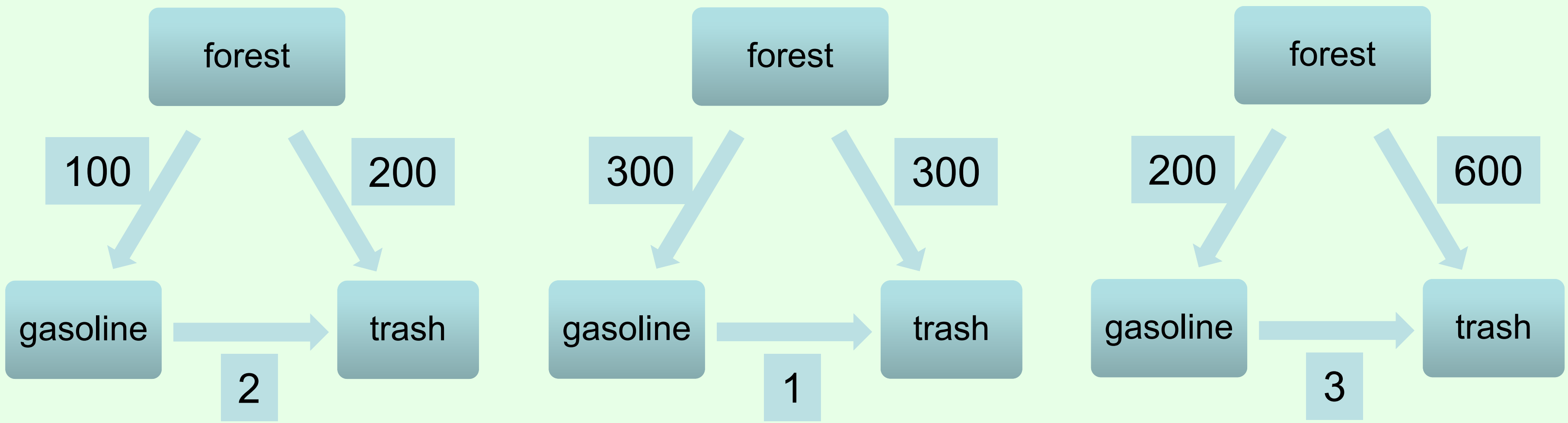
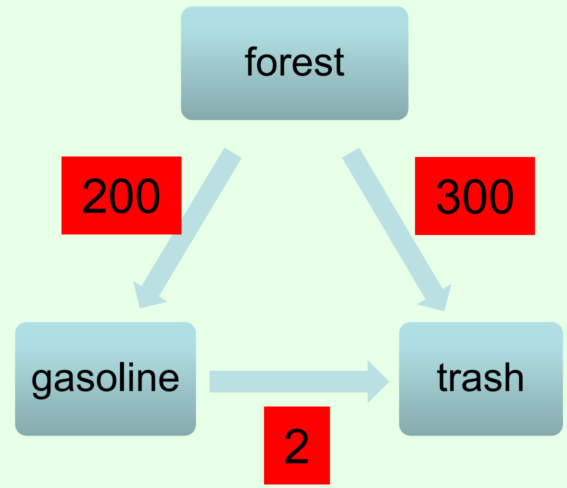
Recall our motivating example



Just taking medians pairwise results in inconsistency



Recall the rule from the midterm

- Let $t_{a,b,i}$ be voter i's tradeoff between a and b
- Tradeoff profile t has score

  $\Sigma_i \Sigma_{a,b} | t_{a,b} - t_{a,b,i} |$
- Upsides:
  - Coincides with median for 2 activities
- Downsides:
  - Dependence on choice of units:

    $| t_{a,b} - t_{a,b,i} | \neq | 2t_{a,b} - 2t_{a,b,i} |$
  - Dependence on direction of edges:

    $| t_{a,b} - t_{a,b,i} | \neq | 1/t_{a,b} - 1/t_{a,b,i} |$
  - We don't have a general algorithm

A generalization

- Let $t_{a,b,i}$ be voter i's tradeoff between a and b
- Let f be a monotone increasing function – say, $f(x) = x^2$
- Tradeoff profile t has score

  $$\Sigma_i \Sigma_{a,b} | f(t_{a,b}) - f(t_{a,b,i}) |$$

- Still coincides with median for 2 activities!

$t_{a,b}$: **1** **2** **3**

$f(t_{a,b})$: **1** **4** **9**

An MLE justification

- Suppose probability of tradeoff profile $\{t_i\}$ given true tradeoff t is

  $\prod_i \prod_{a,b} \exp\{-| f(t_{a,b}) - f(t_{a,b,i}) |\}$

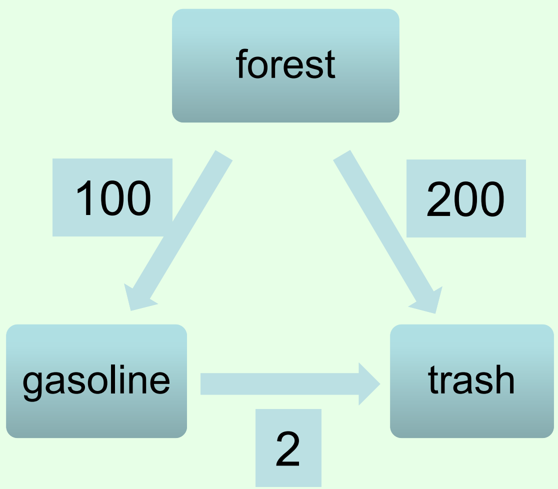
- Then $\arg\max_t \prod_i \prod_{a,b} \exp\{-| f(t_{a,b}) - f(t_{a,b,i}) |\} =$
  $\arg\max_t \log \prod_i \prod_{a,b} \exp\{-| f(t_{a,b}) - f(t_{a,b,i}) |\} =$
  $\arg\max_t \Sigma_i \Sigma_{a,b} -| f(t_{a,b}) - f(t_{a,b,i}) | =$
  $\arg\min_t \Sigma_i \Sigma_{a,b} | f(t_{a,b}) - f(t_{a,b,i}) |$
  which is our rule!

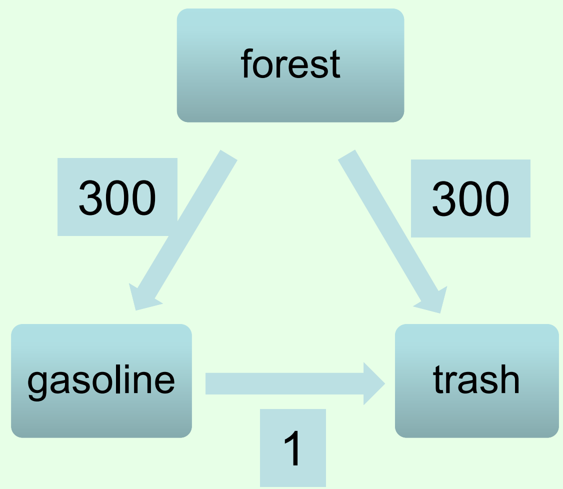
So what's a good f?

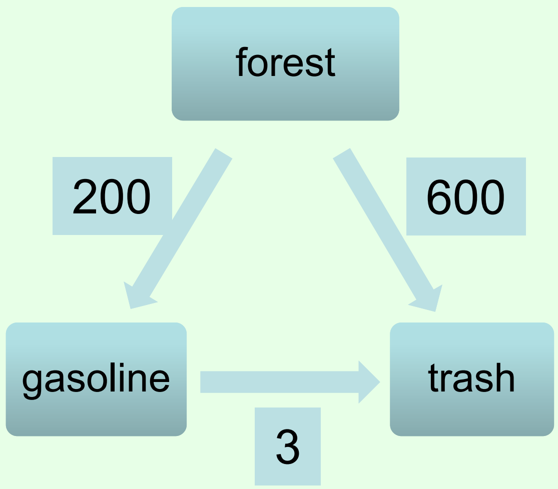
- Intuition: Is the difference between tradeoffs of 1 and 2 the same as between 1000 and 1001, or as between 1000 and 2000?
- So how about $f(x)=\log(x)$?
  - (Say, base e – remember $\log_a(x)=\log_b(x)/\log_b(a)$)

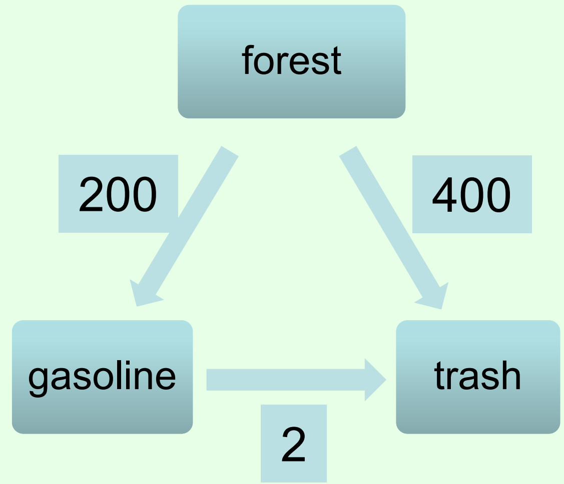
$t_{a,b}$	**1 2**		**1000**	**2000**
$\ln(t_{a,b})$	**ln(1)**	**ln(2)**	**ln(1000)**	**ln(2000)**
	0	0.69	6.91	7.60

On our example









Properties

- Independence of units

  $| \log(1) - \log(2) | = | \log(1/2) | =$

  $| \log(1000/2000) | = | \log(1000) - \log(2000) |$

  More generally:

  $| \log(ax) - \log(ay) | = | \log(x) - \log(y) |$

- Independence of edge direction

  $| \log(x) - \log(y) | = | \log(1/y) - \log(1/x) | =$

  $| \log(1/x) - \log(1/y) |$

Consistency constraint becomes additive

$xy = z$

is equivalent to

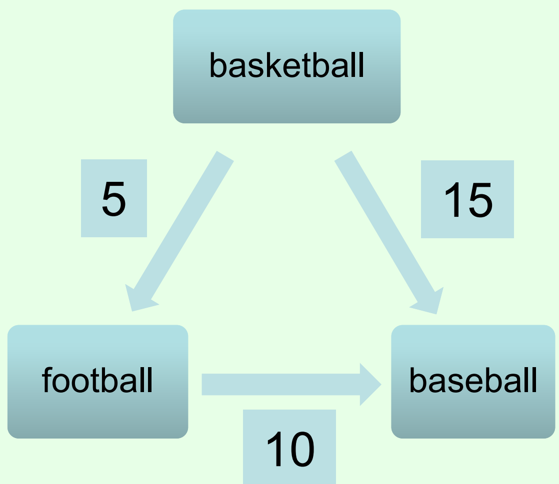
$\log(xy) = \log(z)$

is equivalent to

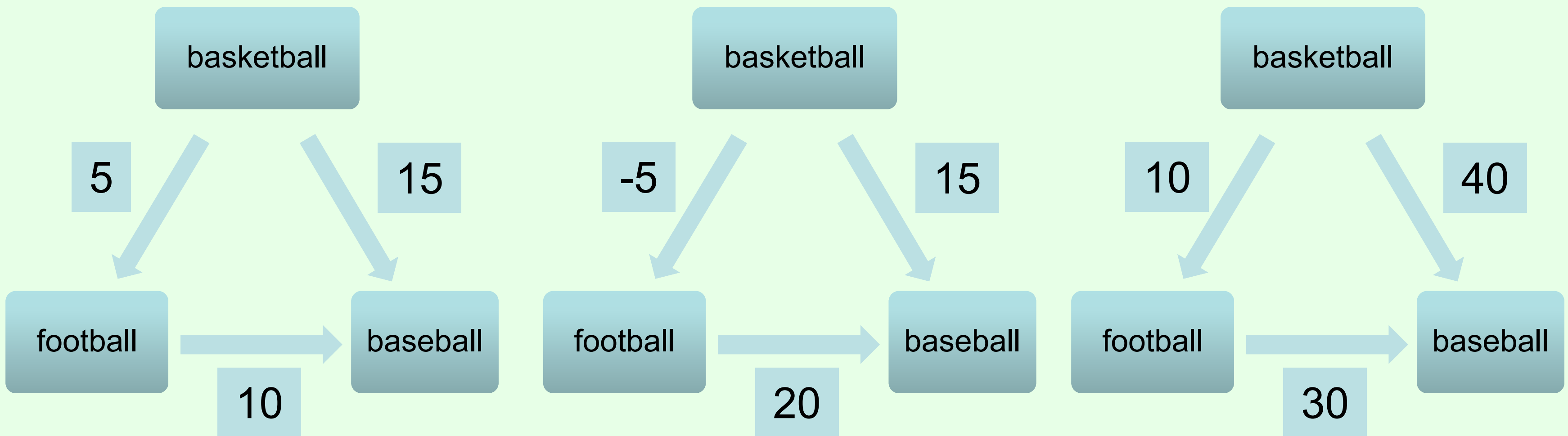
$\log(x) + \log(y) = \log(z)$

An additive variant

- "I think basketball is 5 units more fun than football, which in turn is 10 units more fun than baseball"

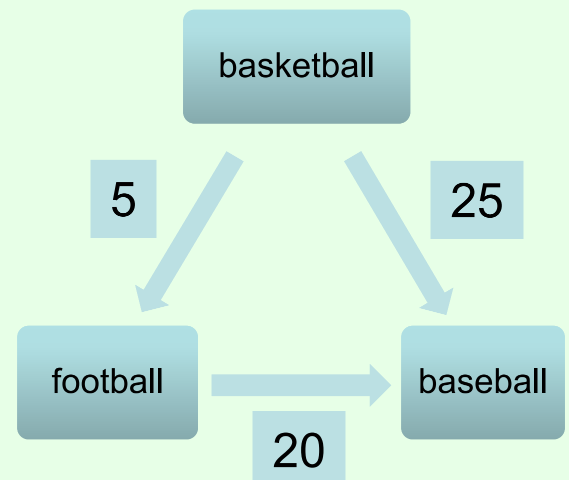


Aggregation in the additive variant



Natural objective:

minimize $\Sigma_i \Sigma_{a,b} d_{a,b,i}$ where $d_{a,b,i} = | t_{a,b} - t_{a,b,i} |$ is the distance between the aggregate difference $t_{a,b}$ and the subjective difference $t_{a,b,i}$



objective value 70 (optimal)

A linear program for the additive variant

q_a: aggregate assessment of quality of activity a (we're really interested in $q_a - q_b = t_{a,b}$)

$d_{a,b,i}$: how far is i's preferred difference $t_{a,b,i}$ from aggregate $q_a - q_b$, i.e., $d_{a,b,i} = |q_a - q_b - t_{a,b,i}|$

minimize $\Sigma_i \Sigma_{a,b} d_{a,b,i}$

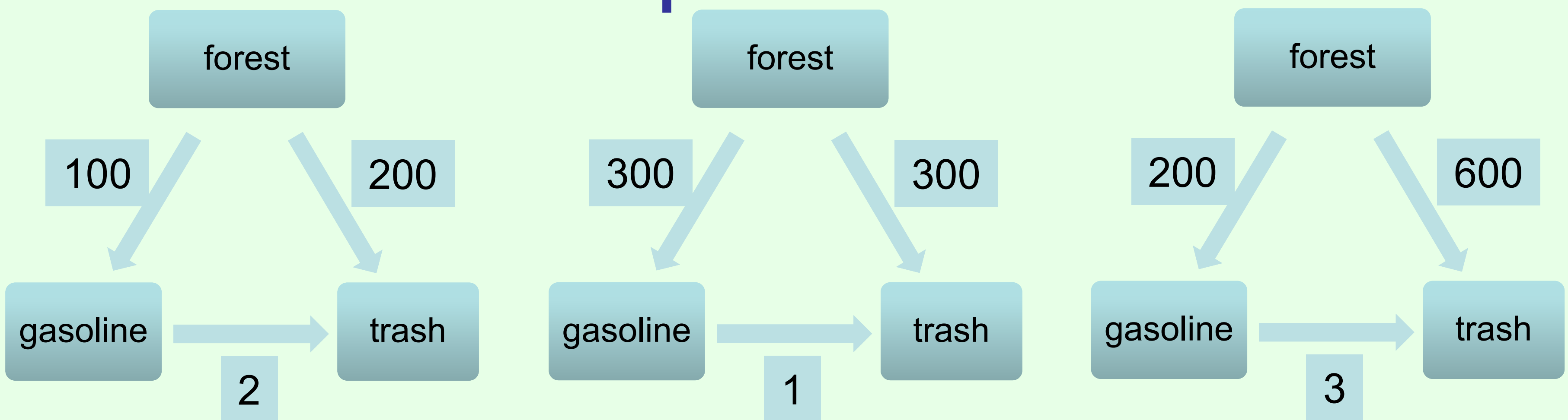
subject to

for all a,b,i: $d_{a,b,i} \geq q_a - q_b - t_{a,b,i}$

for all a,b,i: $d_{a,b,i} \geq t_{a,b,i} - q_a + q_b$

(Can arbitrarily set one of the q variables to 0)

Applying this to the logarithmic rule in the multiplicative variant



Just take logarithms on the edges, solve the additive variant, and exponentiate back