

## Homework 1 (due Feb. 1 before or at the beginning of class)

Note the rules for assignments on the course web page. Show all your work, but circle your final answer. Contact Vince (conitzer@cs.duke.edu) or Michael (malbert@cs.duke.edu) with any questions.

Consider the following scoring rules in a binary setting (the outcome is 1 or 0, and  $p$  indicates the probability of 1):

1.

$$S(p, 1) = \frac{\sqrt{p}}{2} + \frac{1}{2\sqrt{p}}$$

$$S(p, 0) = \frac{\sqrt{p}}{2}$$

2.

$$S(p, 1) = \begin{cases} 1 & \text{if } p > 1/2 \\ 0 & \text{if } p = 1/2 \\ -1 & \text{if } p < 1/2 \end{cases}$$

$$S(p, 0) = \begin{cases} -1 & \text{if } p > 1/2 \\ 0 & \text{if } p = 1/2 \\ 1 & \text{if } p < 1/2 \end{cases}$$

3.

$$S(p, 1) = -2p^3 + 3p^2$$

$$S(p, 0) = -2p^3$$

4.

$$S(p, 1) = p^2 - 2p + 5/4$$

$$S(p, 0) = p^2 - 2p + 1/4$$

Answer the following questions for each of these rules, with proof / explanation in each case.

- Is the rule strictly proper, proper (but not strictly), or not proper? You are encouraged to do this in two different ways.
- Suppose the principal prefers outcome 1 over outcome 0 (and, hence, prefers larger values of  $p$ ). Which of the rules are both proper and principal-aligned?
- For each of the rules, give the maximum amount that the principal might pay if she uses this in a market scoring rule with initial probability  $1/2$ .