Market Scoring Rules

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Multiple forecasters

- What if we have more than one forecaster?
- One idea: use a proper scoring rule for everyone separately, then aggregate
  - Median? Mean? Geometric mean?

A story

- A coin may be biased ($P(H|B) = .75$) or unbiased ($P(H|\text{not } B)=.5$)
- Prior: half of coins are biased ($P(B)=.5$)
- We ask a forecaster to predict the outcome of the next toss of this coin
- Suppose he has seen this coin be tossed once before, and the outcome was H
- What should the forecaster predict (if rewarded with a proper scoring rule?)

Story, continued

- Same as before, but now there are **two** forecasters
- Each of them has observed a separate toss of the coin before, and in both cases the answer was H
- What should they each predict?
  - … assuming they are each rewarded via a proper scoring rule and do not know about each other?
- What should someone who has observed both coin tosses predict?

- … but what if the two forecasters observed the **same** toss?

Takeaway

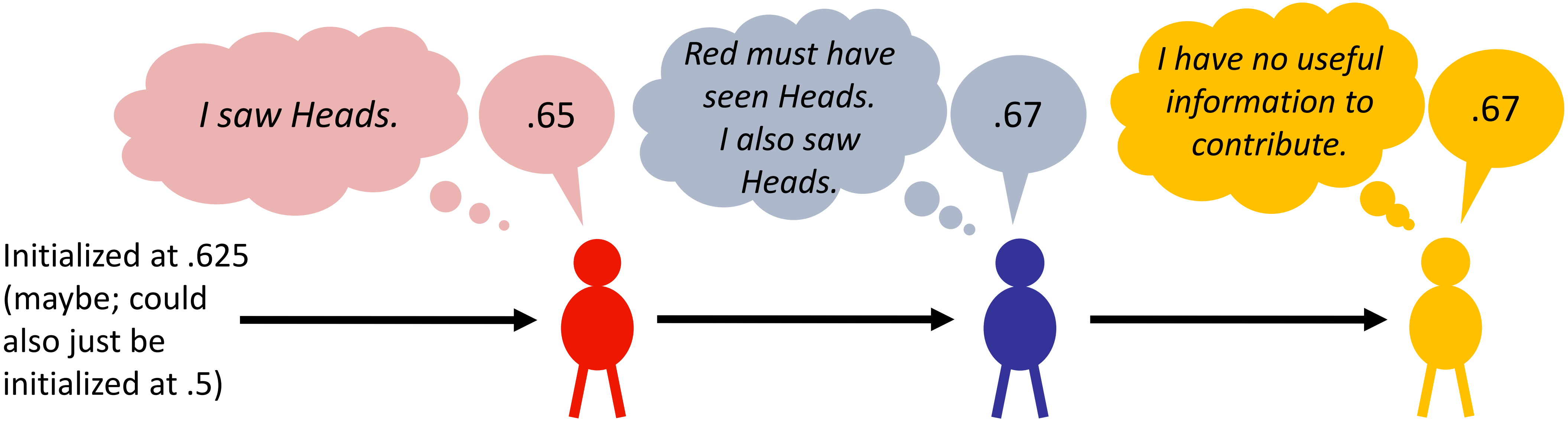
- We would like forecasters' information to **aggregate**
- No way to do this automatically from individual forecasts unless we know more about the information structure
- But what if we want our system to work in a way that is independent of the information structure?
- (Possible) solution: assume the **agents** know the information structure

Market scoring rule

- Let the agents report probabilities $\mathbf{p}_i$ in sequence, i = 1, …, n
- Pay agent i: $S(\mathbf{p}_i, \omega) - S(\mathbf{p}_{i-1}, \omega)$
  - Set $\mathbf{p}_0$ to something arbitrary
- Total payment: $S(\mathbf{p}_n, \omega) - S(\mathbf{p}_0, \omega)$
  - Bounded (if S is bounded)

How it’s supposed to work (with logarithmic scoring rule)

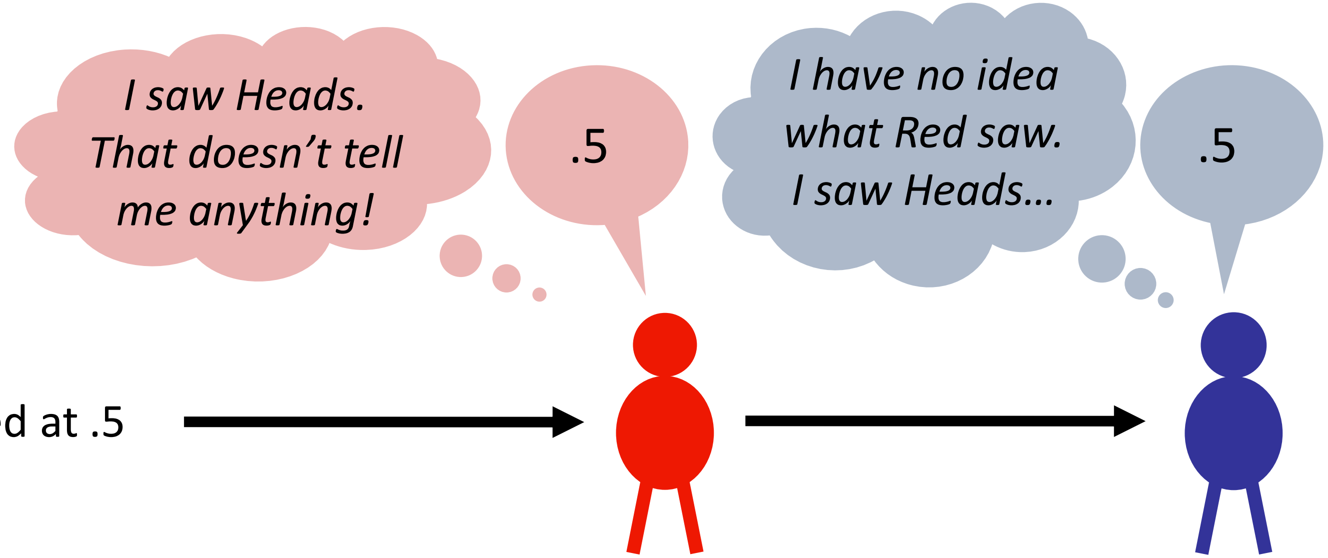
Information has been aggregated!



Payment if Heads happens eventually	ln(.65) - ln(.625)	ln(.67) - ln(.65)	ln(.67) - ln(.67) = 0
Payment if Tails happens eventually	ln(.35) - ln(.375)	ln(.33) - ln(.35)	ln(.33) - ln(.33) = 0

A failure mode: predict XOR of your fair coins

Initialized at .5

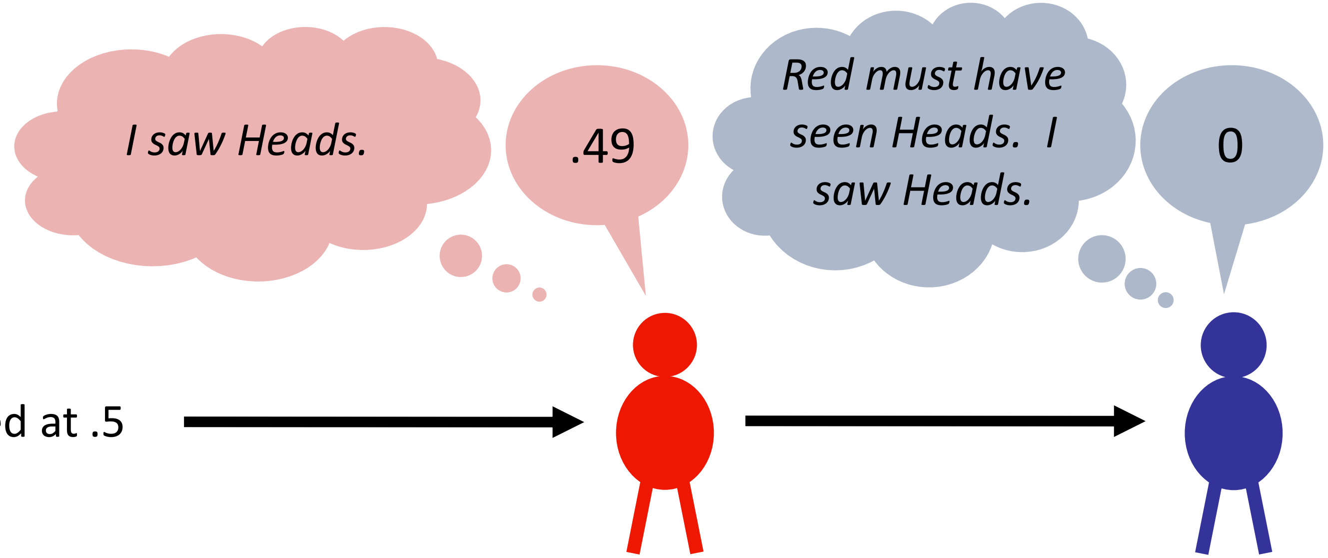


Information has not been aggregated

Brittleness of previous example: suppose coins come up Heads with .51

Information has been aggregated!

Initialized at .5



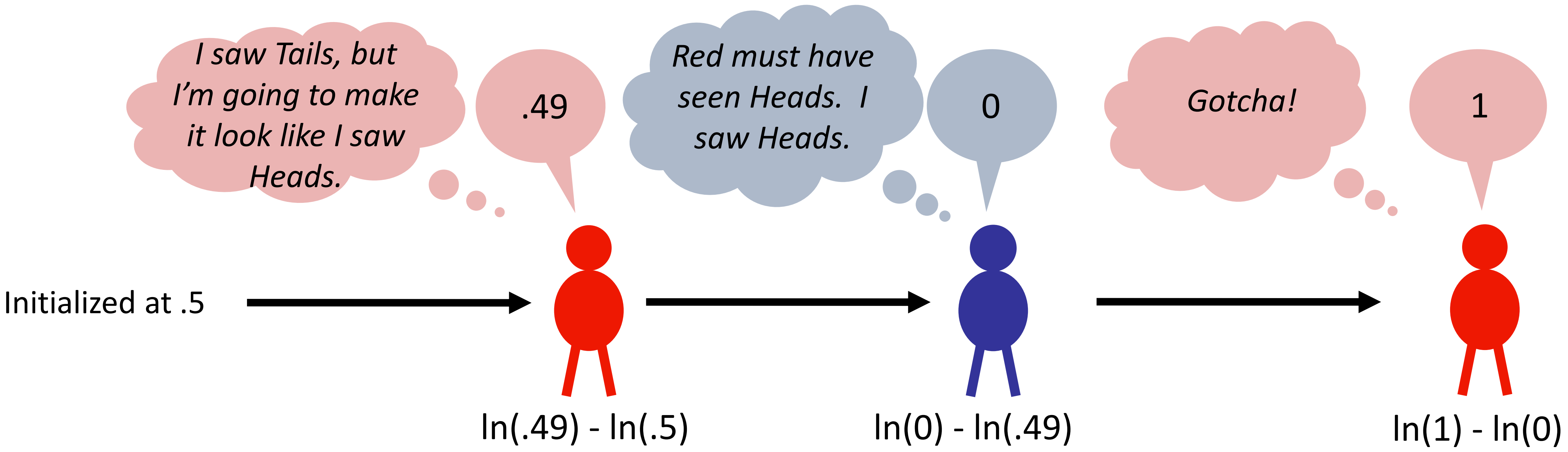
ln(.51) - ln(.5)

ln(1) - ln(.51)

= reported probability that the coins came up the same

Blue takes most of the spoils.
Is this fair?
Incentive to wait? Who will cave first?

Another failure mode: sneaky Red on XOR



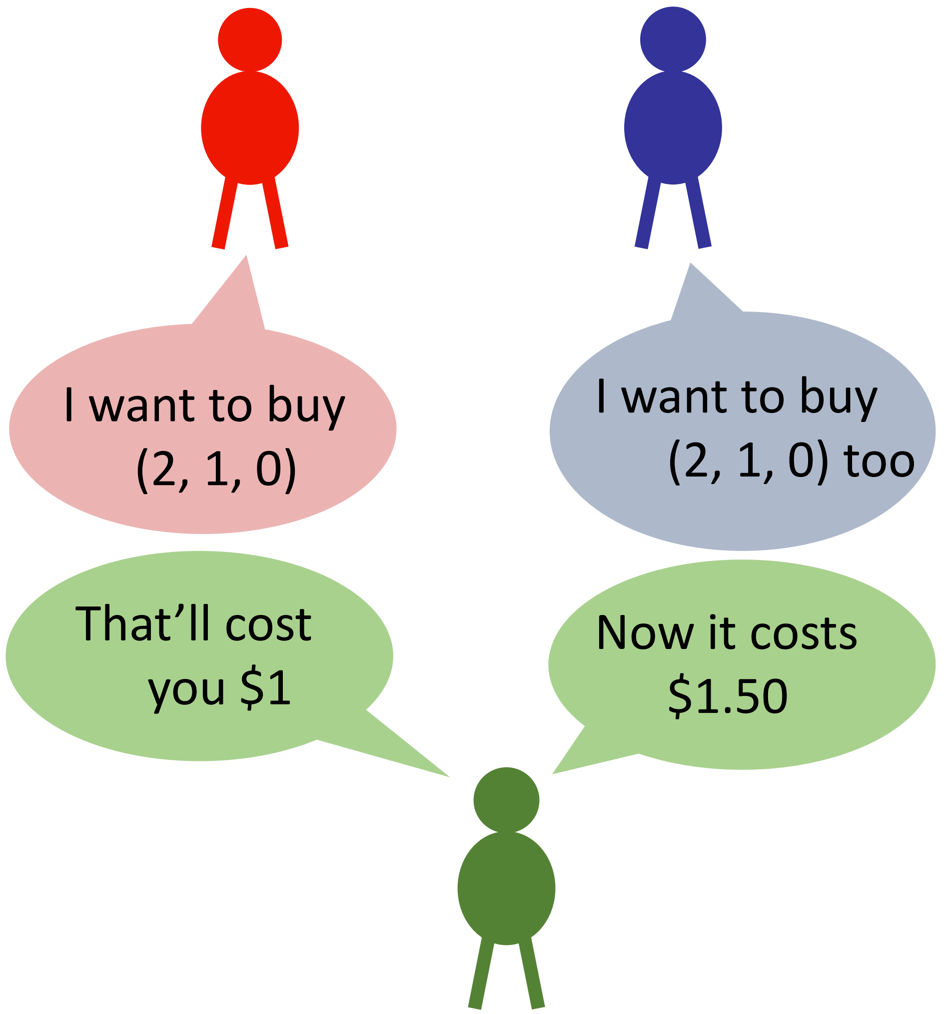
What if Blue is also strategic?

Principal-aligned proper scoring rules, revisited [Shi, Conitzer, Guo 2009]

- Market scoring rules are not principal-aligned (why?)
- Nice thing about market scoring rules: upper bound on subsidy independent of n = #agents providing information
    - I.e., cost of obtaining all information is O(1)
- Upper bound with principal alignment: O(n) (why?)
- Matching lower bound in the paper (assuming nonzero incentive for prediction)

Relationship between market scoring rules and more traditional forms of prediction markets [Hanson 2003, 2007; Chen and Pennock 2007]

- Consider a security that pays out $1 if outcome ω occurs
- Let q_ω denote the number of such securities outstanding
- Consider a market maker always willing to sell more such securities at price $p_\omega(\mathbf{q})$ (or buy at $-p_\omega(\mathbf{q})$)
- Want $\Sigma_\omega\, p_\omega(\mathbf{q}) = 1$ for all $\mathbf{q}$
- Suppose $p_\omega(\mathbf{q})$ is increasing in q_ω (to 1 in the limit)
- Suppose the total paid price is path independent, $C(\mathbf{q})$
  - Moving to $\mathbf{q}'$ from $\mathbf{q}$ costs $C(\mathbf{q}') - C(\mathbf{q})$



Market maker for logarithmic scoring rule

- Let $C(\mathbf{q}) = \ln \Sigma_\omega \exp\{q_\omega\}$
- Then the current price of an ω security is $p_\omega(\mathbf{q}) = \exp\{q_\omega\} / \Sigma_{\omega'} \exp\{q_{\omega'}\}$
- Suppose you believe the probability vector should be $\mathbf{p}'$
- Suppose you end up trading the market towards a $\mathbf{q}'$ such that $\Sigma_{\omega'} \exp\{q'_{\omega'}\} = k$
- You will want to have $\exp\{q'_\omega\} / k = p'_\omega$ or $q'_\omega = \ln kp'_\omega$ (why?)
- So you will pay $\ln \Sigma_{\omega'} \exp\{q'_{\omega'}\} = \ln k$ (minus a constant)
- For the realized outcome ω you will receive $q'_\omega = \ln kp'_\omega$ (minus a constant)
- So you get $\ln kp'_\omega - \ln k = \ln p'_\omega$ (plus/minus a constant)!