

## Programming Assignment 2.5: Estimating Utilities (due April 6 before class)

Please read the rules for assignments on the course web page (<http://www.cs.duke.edu/courses/spring18/compsci223/>). Use Piazza (preferred) or directly contact Harsh ([harsh.parikh@duke.edu](mailto:harsh.parikh@duke.edu)), Hanrui ([hrzhang@cs.duke.edu](mailto:hrzhang@cs.duke.edu)), or Vince ([conitzer@cs.duke.edu](mailto:conitzer@cs.duke.edu)) with any questions. Please use clear variable names and write comments in your code where appropriate (you can put comments between `/*` and `*/`, or start a line with `#`).

Please see Homework 1 for details about getting set up with GLPK, making a directory for this homework, etc.

Note: in this assignment, there is an example instance that you are asked to solve. However, just getting this example right is *not* enough to get full credit: your formulation should work on all instances. The example is just there to give you something to test your formulation on.

### 1. (Estimating utilities (50 points).)

Elena Umberta Massima is an expected utility maximizer. When presented with two probability distributions over a set of possible outcomes, E.U.M. says, without hesitation, which she prefers, and you will not catch her in any inconsistencies.

We have four outcomes:  $A, B, C, D$ . We will accordingly represent probability distributions as vectors of four probabilities of the respective outcomes.  $(p_A, p_B, p_C, p_D) \succ (p'_A, p'_B, p'_C, p'_D)$  will denote that E.U.M. prefers distribution  $p$  to  $p'$ . We learn the following four preferences:

- $(.1, .2, .3, .4) \succ (.1, .2, .4, .3)$
- $(.4, .4, .1, .1) \succ (.4, .2, .2, .2)$
- $(.6, .1, 0, .3) \succ (.4, .3, .3, 0)$
- $(.4, .3, .2, .1) \succ (.5, .5, 0, 0)$

Obviously, we jump on the opportunity to estimate the utilities (for outcomes  $A, B, C, D$ ) of this fascinating woman.

Our goal will be to assign utilities in the interval  $[0, 1]$  to the four outcomes that are consistent with E.U.M.'s preferences. Write a linear program formulation for this. You should add an objective to satisfy the consistency constraints by as large a margin as possible (similar to our linear program for strict dominance by mixed strategies). You should use the MathProg (`.mod`) language to model the general problem (you should allow for more than four outcomes and more than four preferences) and solve the specific instance above. (Hint: the optimal objective value is 0.02.)