Recall the properties of relations on sets (where $R$ is the relation on set $A$):

- **Reflexive**: $\forall a \in A. aRa$.
- **Irreflexive**: $\forall a \in A. \neg(aRa)$.
- **Transitive**: $\forall a, b, c \in A. aRb \land bRc \rightarrow aRc$.
- **Symmetric**: $\forall a, b \in A. aRb \rightarrow bRa$.
- **Asymmetric**: $\forall a, b \in A. aRb \rightarrow \neg(bRa)$.
- **Antisymmetric**: $\forall a, b \in A. aRb \land bRa \rightarrow a = b$.

1. Prove by ordinary induction that, for any natural number $n$, $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$.

2. Prove by ordinary induction that, for any natural number $n$, $3^{n+2} + 2^{4n+2}$ is divisible by 13. [Hint: Does this look familiar? How can you leverage your proof that used the WOP?]

3. Consider the relation $R$ on $\{1, 2, 3, 4, 5\}$ is an equivalence relation:

   $$R = \{(1, 1), (1, 4), (4, 1), (4, 4), (5, 5), (2, 2), (2, 3), (3, 2), (3, 3)\}$$

   (a) Verify $R$ is an equivalence relation.
   (b) What is $[3]$?
   (c) What is the partition induced by $R$?

4. Consider the relation $R$ on set $A = \{n \in \mathbb{Z} \mid 1 \leq n \leq 10\}$:

   $$R = \{(x, y) \in A \times A \mid x = y \lor (x \text{ is odd} \land x < y)\}$$

   (a) Verify that $R$ is a partial order.
   (b) What is the size of the largest chain in $R$?
   (c) What is the size of the largest antichain in $R$?
   (d) At least how many chains must any chain decomposition of $R$ have?

5. Let $A$ be a set and let $R$ be a relation on $A$.

   (a) Prove or disprove that there exists an equivalence relation $S$ on $A$ such that $R \subseteq S$.
   (b) Prove or disprove that if $R$ is reflexive, there exists a partial order $S$ on $A$ such that $S \subseteq R$. 

1