- vertices and edges
  - $n$: # of vertices
  - $m$: # of edges
  - Often: in a graph, the same edge only appears at most once. In this case $m \leq n^2$. Vertices in a graph are connected. In this case $m \geq n - 1$.

- Representing graphs
  - Adjacency array
    
    $O(n^2)$ space, can check $(i, j)$ is an edge $O(1)$
    
    Enumerate edges adjacent to a vertex $O(n)$
    
    Better for dense graphs when $m = \Theta(n^2)$

  - Adjacency list
    
    $O(n + m)$, check if $(i, j)$ is an edge $O(\text{degree}(i))$
    
    Enumerate edges adjacent to $i$ $O(\text{degree}(i))$

- Definition: degree of a vertex, $\text{degree}(i)$ is the number of edges that is adjacent to $i$.

  (Directed) in-degree of a vertex, $\text{in-deg}(i)$ is # incoming edges

  Out-degree of a vertex, $\text{out-deg}(i)$ is # outgoing edges.

- Claim: For an undirected graph, $\sum_{i \in V} \text{degree}(i) = 2m$.

  $\sum_{i \in V} \text{degree}(i) = 2m$.
if the graph is sparse \( m = \Theta(n) \)

\[
\text{average degree} \quad \frac{\sum \text{degree}(i)}{n} = \frac{2m}{n} = \Theta(1).
\]

- DFS

\[
\text{DFS\_visit (1)} \rightarrow \text{DFS\_visit (2)} \rightarrow \text{DFS\_visit (3)} \rightarrow \text{DFS\_visit (4)} \rightarrow \text{DFS\_visit (5)}
\]

- DFS tree

- DFS tree is not unique
  
  can choose different starting points and/or the ordering of edges

  decide to follow 
  
  (1,3) before (1,2)

  (1,4) first.

  Cannot be a DFS tree if 1 is the root (starting point)

- pre-order and post-order
- **Pre-order and post-order**

```
- Pre-order: 1 2 4 3 6
- Post-order: 3 4 6 2 1
```

- Pre-order: orderly that draw the vertices.
- Post-order: orderly in which the subtrees are finished.