Ch. 7 - Pushdown Automata

A DFA = \((Q, \Sigma, \delta, q_0, F)\)
Modify DFA by adding a stack. New machine is called Pushdown Automata (PDA).
Definition: Nondeterministic PDA (NPDA) is defined by

\[ M=(Q, \Sigma, \Gamma, \delta, q_0, z, F) \]

where

- \( Q \) is finite set of states
- \( \Sigma \) is tape (input) alphabet
- \( \Gamma \) is stack alphabet
- \( q_0 \) is initial state
- \( z \) - start stack symbol, \( z \in \Gamma \)
- \( F \subseteq Q \) is set of final states.
- \( \delta:Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \rightarrow \text{finite subsets of } Q \times \Gamma^* \)
Example of transitions

\[ \delta(q_1,a,b) = \{(q_3,b),(q_4,ab),(q_6,\lambda)\} \]

The diagram for the above transitions is:
Instantaneous Description:

\[(q,w,u)\]

Description of a Move:

\[(q_1,aw,bx) \vdash (q_2,w,yx)\]

iff

\[\left( q_2, y \right) \in \delta(q_0, a, b) \]

Definition Let \( M=(Q,\Sigma,\Gamma,\delta,q_0,z,F) \) be a NPDA. \( L(M) = \{ w \in \Sigma^* \mid (q_0,w,z) \vdash^* (p,\lambda,u), p \in F, u \in \Gamma^* \} \). The NPDA accepts all strings that start in \( q_0 \) and end in a final state.
Example: $L = \{a^n b^n | n \geq 0\}$, $\Sigma = \{a, b\}$, $\Gamma = \{z, a\}$
Another Definition for Language Acceptance

NPDA $M$ accepts $L(M)$ by empty stack:

$$L(M) = \{ w \in \Sigma^* \mid (q_0, w, z)^* \vdash (p, \lambda, \lambda) \}$$
Example: $L = \{a^n b^m c^{n+m} | n, m > 0\}$, 
$\Sigma = \{a, b, c\}$, $\Gamma = \{0, z\}$
Examples for you to try on your own: (solutions are at the end of the handout).

- \( L = \{ a^n b^m | m > n, m, n > 0 \}, \Sigma = \{ a, b \}, \Gamma = \{ z, a \} \)
- \( L = \{ a^n b^{n+m} c^m | n, m > 0 \}, \Sigma = \{ a, b, c \} \)
- \( L = \{ a^n b^{2n} | n > 0 \}, \Sigma = \{ a, b \} \)
Definition: A PDA
\( M=(Q, \Sigma, \Gamma, \delta, q_0, z, F) \) is deterministic if for every \( q \in Q, \ a \in \Sigma \cup \{ \lambda \}, \ b \in \Gamma \)

1. \( \delta(q, a, b) \) contains at most 1 element
2. if \( \delta(q, \lambda, b) \neq \emptyset \) then \( \delta(q, c, b) = \emptyset \) for all \( c \in \Sigma \)

Definition: \( L \) is DCFL iff \( \exists \) DPDA \( M \) s.t. \( L = L(M) \).
Examples:

1. Previous pda for $\{a^n b^n | n \geq 0\}$ is deterministic?

2. Previous pda for $\{a^n b^m c^{n+m} | n, m > 0\}$ is deterministic?

3. Previous pda for $\{w w^R | w \in \Sigma^+\}, \Sigma = \{a, b\}$ is deterministic?
Example: \( L = \{ a^n b^m | m > n, m, n > 0 \} \), \( \Sigma = \{ a, b \} \), \( \Gamma = \{ z, a \} \)

Example: \( L = \{ a^n b^{n+m} c^m | n, m > 0 \} \), \( \Sigma = \{ a, b, c \} \),

Example: \( L = \{ a^n b^{2n} | n > 0 \}, \Sigma = \{ a, b \} \)